Test 3 Review

Chapter 6: The Laplace Transform

6.1:
The Laplace transform of \( f(t) \) is defined through an improper integral:

\[
\int_0^{\infty} f(t) e^{-st} \, dt.
\]

Be able to calculate the transform of all the basic functions, given in the table on page 319 (see Examples 4,5,6 and 7, p.311-312 and assigned hw).
Remember that the Laplace transform does not exist for all functions. The functions need to be piecewise continuous and of exponential order for the Laplace transform to exist.

6.2:
- Know how to transform derivatives of functions:
  (1) \( L\{y\}' = sL\{y\} - y(0) \) and
  (2) \( L\{y''\} = s^2L\{y\} - sy(0) - y'(0) \) (entry 18 in the Table)
- Know how to compute inverse transform functions. You will have to use some algebraic manipulation and/or partial fractions decomposition (PFD) to put it in a form that can be found in the Table 6.2.1. (Exercises 1-10)
- Know how to solve linear differential equations using the Laplace transform. This involves three steps:
  - Apply the Laplace transform to both sides of the equation and use formulas (1) and (2) above so that the equation contains the Laplace of \( y \) only (which we denote by \( Y(s) \)).
  - Plug in the initial conditions.
  - Solve for \( Y(s) \).
  - Compute the inverse transform of \( Y(s) \). This is the solution to the DE. (Exercises 11-23)
- Know how to use the formula for the derivative of the Laplace:
  \[
  F^{(n)}(s) = L\{(-t)^n f(t)\}.
  \]
  (entry 19 in the Table) (see Exercises 29-34))

6.3:
- Know the definition of the unit step function \( u_c(t) \). (Example 1)
- Know how to write a piecewise function in terms of the unit step functions and use entries 12 and 13 in the Table 6.2.1 to find the Laplace transform (Exercises 7-12 and 25, Example 2). Conversely, know how to use entry 13 to find the inverse Laplace of functions of the form \( e^{-cs} F(s) \) (Exercises 14-18, Example 3)
- Know the translation property for the Laplace transform (entry 14 in the Table). This property is useful because it can avoid using partial fraction decomposition.(Exercise 17, Example 4)

6.4:
Know how to apply the properties of the unit step function \( u_c(t) \) learned in Section 6.3 to solve differential equations with discontinuous \( g(t) \). (Examples 1 and 2, Exercises 1-12).

6.5:
- Know the definition and the properties of the unit impulse function. In particular, remember the property:
  \[
  \delta(t-c)f(t) = \delta(t-c)f(c)
  \]
  from which it follows:
  \[
  L\{\delta(t-c)f(t)\} = e^{-sc}f(c)
  \]
  (this formula is not included in the table)
- Know how to solve differential equations with \( g(t) \) involving delta functions and unit step functions (Example 1, Exercises 1-11)

6.6:
- Know the definition of the convolution integral:
  \[
  f \ast g = \int_0^t f(t-\tau)g(\tau) \, d\tau.
  \]
  Practice evaluating some easy convolution integrals such as the one in Exercise 6. Know how to find the Laplace transform of convolution integrals (entry 16 in the Table) (Exercises 4-7).
- The convolution product offers an alternative to using PFD for determining the inverse of a product: given \( H(s) = F(s)G(s) \), we can find the inverse by evaluating \( f(t) \ast g(t) \) where \( f \) and \( g \) are the inverse transforms of \( F \) and \( G \) respectively (Exercises 8-11).
- Practice solving DE using the convolution integral (Exercises 14-16) but keep in mind you might be asked to actually evaluate the integral.