ANSWERS TO EVEN HW PROBLEM CHAPTER 16

Section 16.1.
16. corresponds to Graph I, since the horizontal vector components remain constant, but the vectors above the xy-plane point generally upward, while the vectors below the xy-plane point generally downward.
18. corresponds to Graph II; each vector $\mathbf{F}(x, y, z)$ has the same length and direction as the position vector at the point $(x, y, z)$, and therefore the vectors all point directly away from the origin.

Section 16.2
18. Vectors starting on $C_1$ point in roughly the same direction as $C_1$, so the line integral is positive. On the other hand, no vectors starting on $C_2$ point in the same direction as $C_2$, while some vectors point in roughly the opposite direction, so we would expect the line integral to be negative.

38. The parabola can be parametrized by $\mathbf{r}(t) = \langle t, t^2 \rangle$ with $-1 \leq t \leq 2$. We have:
$$\int_C \mathbf{F} \cdot d\mathbf{r} = \int_C \mathbf{F}(\mathbf{r}(t)) \cdot \mathbf{r}'(t) \, dt = \int_{-1}^{2} \langle t \sin(t^2), t^2, -1 \rangle \cdot \langle 1, 2t, 0 \rangle \, dt = \int_{-1}^{2} t \sin(t^2) + 2t^3 \, dt = \frac{1}{2} \cos(1) + 15 - \cos(4)$$

Section 16.3: 
2. $\int_C \nabla \cdot d\mathbf{r} = f(\mathbf{r}(1)) - f(\mathbf{r}(0)) = f(2, 2) - f(1, 0) = 9 - 3 = 6$

Section 16.4: 
4. 0 12. 0 14. -16 18. 12$\pi$

Section 16.5:
10. (a) divergence is positive (b) curl is the zero vector.
12. (a) meaningless because $f$ is a scalar (b) vector field (c) scalar field (d) vector field (e) meaningless because $\mathbf{F}$ is not a scalar field (f) vector field (g) scalar field (h) meaningless because $f$ is a scalar field (i) vector field (j) meaningless because $\div \mathbf{F}$ is a scalar field (k) meaningless because div $\mathbf{F}$ is a scalar field (l) scalar field.

Section 16.6
12. We have: $x^2 + y^2 = u^2 \cos^2 v + u^2 \sin^2 v = u^2 = z^2$, which represents the equation of a cone with axis the x-axis, graph V.
14. We have $y^2 + z^2 = u^2$ so, if $u$ is held constant, each grid curve is a circle of radius $u$ in the plane $x = u^3$. The graph then must be graph III.
16. These equations correspond to graph VI: when $u = 0$, then $x = 3 + \cos v$, $y = 0$, and $z = \sin v$, which are equations of a circle with radius 1 in the $xz$-plane centered at $(3,0,0)$. When $u = \frac{1}{2}$, then $x = \frac{3}{2} + \frac{1}{2} \cos v$, $y = 0$ and $z = \frac{3}{2} + \frac{1}{2} \sin v$, which are equations of a circle with radius $\frac{1}{2}$ in the $xz$-plane centered at $\left(\frac{3}{2}, 0, \frac{3}{2}\right)$. This suggests that the grid curves with $u$ constant are the vertically oriented circles visible on the surface. The spiraling grid curves correspond to keeping $v$ constant.
20. $x = 4 - y^2 - 2z^2, y = y, z = z$ where $y^2 + 2z^2 \leq 4$ since $x \geq 0$. Then the associated vector equation is $\mathbf{r}(y, z) = \langle 4 - y^2 - 2z^2 \rangle \mathbf{i} + y \mathbf{j} + z \mathbf{k}$.
22. In spherical coordinates, parametric equations are $x = 4 \sin \phi \cos \theta$, $y = 4 \sin \phi \sin \theta$, $z = 4 \cos \phi$.

The intersection of the sphere with the plane $z = 2$ corresponds to $z = 4 \cos \phi = 2 \Rightarrow \cos \phi = \frac{1}{2} \Rightarrow \phi = \frac{\pi}{3}$. By symmetry, the intersection of the sphere with the plane $z = -2$ corresponds to $\phi = \pi - \frac{\pi}{3} - \frac{2\pi}{3}$. Thus the surface is described by $0 \leq \theta \leq 2\pi, \frac{\pi}{3} \leq \phi \leq \frac{2\pi}{3}$. 
Section 16.7:

10. Using the "natural" parametrization: \( x = y + 2z^2, \ y = y, \ z = z \), with domain \( D : 0 \leq y \leq 1, \ 0 \leq z \leq 1 \).

   We have \( |\vec{r}_y \times \vec{r}_z| = \sqrt{2 + 16z^2} \) and \( \iint_S z^2 \sqrt{2 + 16z^2} dydz = \frac{13}{12}\sqrt{2} \).

12. The surface is made up of three parts: \( S_1 \), the lateral surface of the cylinder, \( S_2 \) the front of the cylinder (\( S_2 \) is the ellipse cut out from the plane \( x + y = 2 \) by the cylinder) and \( S_3 \), the back of the cylinder in the plane \( y = 0 \).

   \( S_1 \) can be parametrized by \( x = \cos(\theta), \ y = y, \ z = \sin(\theta) \) with domain \( D: 0 \leq y \leq 2 - \cos(\theta), \ 0 \leq \theta \leq 2\pi \).

   We have \( |\vec{r}_\theta \times \vec{r}_y| = 1 \) so that \( \iint_{S_1} xy dS = \int_0^{2\pi} \int_0^{2 - \cos(\theta)} \cos(\theta) y dy d\theta = -2\pi \).

   \( S_2 \) can be parametrized by \( x = x, \ y = 2 - x, \ z = z \) with domain \( D: x^2 + z^2 \leq 1 \).

   We have \( |\vec{r}_x \times \vec{r}_z| = \sqrt{2} \) so that
   \[
   \iint_{S_2} xy dS = \iiint_D x(2 - x)\sqrt{2} dA = \sqrt{2} \int_0^{2\pi} \int_0^1 (2r\cos(\theta) - r^2\cos^2(\theta)) r dr d\theta = -\frac{\sqrt{2}}{4} \pi.
   \]

   On \( S_3 \) we have \( y = 0 \), thus \( \iint_{S_3} xy dS = \iint_{S_3} 0 dS = 0 \).

   Adding up the three integrals we have that the integral over \( S \) is given by \( -2\pi - \frac{\sqrt{2}}{4} \pi = -\frac{1}{4}(8 + \sqrt{2})\pi \).

28. We have \( \vec{r}_u \times \vec{r}_v = (\sin v, -\cos v, u) \) and \( \vec{F} \cdot (\vec{r}(u, v)) = (u \sin v, u \cos v, v^2) \) so that
   \[
   \iint_S \vec{F} \cdot d\vec{S} = \int_0^1 \int_0^\pi (u \sin v, u \cos v, v^2) \cdot (\sin v, -\cos v, u) dv du = \int_0^1 \int_0^\pi u \sin^2 v - u \cos^2 v + uv^2 dv du = \frac{\pi^3}{6}.
   \]