11.1 Know how to find the limit of a sequence, how to check whether a sequence is increasing, decreasing or not monotonic and whether the sequence is bounded. Know how to use the squeeze theorem (this theorem is particularly useful for sequences involving sine and cosine functions) and remember that if the sequence of the absolute values converges, then the original sequence converges as well. Read Examples 1,2,4,5,6,7,8,9,10,11.

11.2 Understand the definition of a series and the partial sums of a series. Know how to recognize a geometric series and how to find its sum if the series is convergent. Know how to find the sum of telescopic series. Be familiar with the Test for Divergence. Read examples 1,2,3,4,5,6,8,9.

11.3 Know the statement of the integral test and how to apply it. Remember that this test can be applied only to series with positive terms. Be familiar with p-series, remember that these converge if $p>1$ and diverge otherwise. Know how to estimate the error in approximating a sum of a series with its $n^{th}$ partial sum. Read all examples in the textbook.

11.4: Know the statement of the comparison test and the limit comparison test. Remember that these tests can be applied only to series with positive terms. The series you will compare to are $p$-series or geometric series. Use the comparison test when the inequality is obvious (especially for series involving sine and cosine) and use the limit comparison test in the other cases. In order to determine the series to compare with, you should always start by determining the dominant part of numerator and denominator and neglect the other terms. Remember that, when you use the comparison test to prove divergence you need to find a divergent series which is smaller than your series and when proving convergence you need to find a convergent series which is bigger than your series. Read Examples 1,2,3,4.

11.5: Know the definition of an Alternating Series and the Alternating Series Test. Remember that the Alternating series test is a sufficient but not necessary condition for the convergence of the series, that is, the series can still converge even if condition (i) is not satisfied. If condition (ii) is not satisfied then the series diverges by the Divergence Test. Know how to use the Alternating Series Estimation Theorem to estimate the error in approximating the series with its $n^{th}$ partial sum. Read all the examples in the textbook.

11.6: Know when a series is Absolutely convergent and when it is Conditionally convergent. Remember that absolute convergence implies convergence thus, when asked to test a series for AC or CC you should always start by checking AC. The ratio test is a very convenient tool for checking Absolute convergence. Make sure you know the statement of this test and how to use it (you need to be able to simplify the ratio to be able to take its limit). The ratio test is useful for series involving exponentials and/or factorials. If the series does not contain exponential functions or factorials then the ratio test will most likely be inconclusive and some other test needs to be used. Read Examples 1,2,3,4 in the textbook.

11.8: Know the definition of a power series centered at zero and, more generally, a power series centered at $a$. Use the ratio test to find the interval of convergence and the radius of convergence of a power series. Remember that there are only three possibilities:
1. The series converges on an interval and the radius of convergence is a nonzero number (see Examples 2, 4, 5). Don't forget to check the endpoints of the interval by some test other than the ratio test.
2. The series converges for all $x$ (this happens when the ratio test gives a limit zero for every $x$, See Example 3).
3. The series converges only at $x = a$ (this happens when the ratio test gives a limit of infinity for every $x$ except $x = a$; see Example 1).
Read all Examples in the textbook.
11.9: Know how to manipulate the geometric series \( \sum_{n=0}^{\infty} x^n = \frac{1}{x-1} \) to find power series for related functions and their radius of convergence (Examples 1, 2, 3). This includes differentiation and integration of the power series (Examples 4, 5, 6, 7). Know how to use a power series to estimate the value of a definite integral (Example 8). Read all the examples in the textbook.

11.10: Know how to find the Taylor and/or Maclaurin series of a function. Be familiar with the Maclaurin series for \( e^x, \sin x, \cos x, \arctan x \). Know how to use known Taylor series to find Taylor series for related functions (see Example 6). Know how to use a Taylor series to approximate definite integrals and to find the limits (Examples 8 and 9). Read Examples 1, 3, 4, 5, 6, 7, 8, 9 in the textbook.

Recommended review problems:
Review all homework and notes.
CHAPTER 11 REVIEW:
CONCEPT CHECK: 1, 2, 3, 4, 5(not (g)), 6, 7(a), (c), 8, 9, 10(a)(b)(c), 11
EXERCISES: 1-8, 11-17, 20, 21, 23-26, 27, 28, 29, 32, 35, 36, 40-46, 47-52, 56, 59

ANSWERS to even EXERCISES from Chapter 11 Review:

2. convergent, the limit is 0.
4. \( \cos \left( \frac{n \pi}{2} \right) \) = \{0, −1, 0, 1, 0, −1, ...\} thus the limit does not exists
6. convergent to 0 (by l'Hospital) 8. convergent to 0 12. divergent by limit comparison with 1/n 14. convergent by AST 16. divergent by divergence test. 20. divergent by Ratio Test 24. AC (p-series)
26. divergent by divergence test 28: 11/18 32: \( \frac{416.909}{99.900} \)

36: (a) \( s_3 \approx 1.017305 \) \( R_3 \leq \int_{\infty}^{\infty} \frac{dx}{x} = 0.000064 \) so the error is at most 0.000064
(b) \( R_9 \leq \int_{\infty}^{\infty} \frac{dx}{x^6} = \frac{1}{5 \cdot 9^5} \approx 3.4 \times 10^{-6} \) so to five decimal places
\( \sum_{n=1}^{\infty} \frac{1}{n^3} \approx s_9 \approx 1.01734 \)
40: \( I = [-5,5] \) 42: \( I = (-\infty, \infty) \)

44: \( R = 1/4 \)
46: \( \frac{1}{2} - \frac{\sqrt{3}}{2} \left( x - \frac{\pi}{3} \right) - \frac{1}{4} \left( x - \frac{\pi}{3} \right)^2 + \frac{\sqrt{3}}{12} \left( x - \frac{\pi}{3} \right)^3 + \frac{1}{48} \left( x - \frac{\pi}{3} \right)^4 + ... \)
48: \( \tan^{-1}(x^2) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{4n+2}}{2n+1}, R = 1 \)
50: \( xe^{2x} = \sum_{n=0}^{\infty} \frac{2^n x^{n+1}}{n!}, R = \infty \)
52: \( 10^x = e^{(\ln 10)x} = \sum_{n=0}^{\infty} \frac{(\ln 10)^n x^n}{n!}, R = \infty \)
56: \( \int_{0}^{1} (1 + x^4)^{1/2} \, dx \approx 1.09 \) correct to two decimal places (the error can be determined by the Alternating Series Estimation Theorem)