Intersection of the surface $z = f(x,y)$ with the plane $y = b$.

$f_x(a,b)$ is the slope of the tangent line to the surface at $(a,b,f(a,b))$ in the direction of the positive x-axis.
The direction vector of the tangent line is given by $<1, 0, f_x(a,b)>$

Intersection of the surface $z = f(x,y)$ with the plane $x = a$.

$f_y(a,b)$ is the slope of the tangent line to the surface at $(a,b, f(a,b))$ in the direction of the positive y-axis.
The direction vector of the tangent line is given by $<0, 1, f_y(a,b)>$