Section 4.1: Maximum and Minimum Values

• Know the Hospital Rule and understand that it can be used only for indeterminate forms of the type $0/0$ (Example 1, 4, WebWorK 1-5, 7, 9) or $\infty/\infty$ (Example 2, 3, WebWorK 4, 6, 8). Do not blindly use the Hospital rule every time you have a quotient. Always check first if the quotient is of the form $0/0$ or $\infty/\infty$ (Example 5, WebWorK 11a).

• In order to use the Hospital rule for the indeterminate forms $0/0$, $(\text{WebWorK} 10, 11b), 14\) or $\infty/\infty$, $(\text{WebWorK} 12, 13)$ you will have first to do some algebraic manipulation in order to put it in the form $0/0$ or $\infty/\infty$ and then use the Hospital (Example 6, 7, 8, 9).

• Know how to prove the special limits: $\lim_{x \to \infty} e^x/x^n = \infty$ for any $n$, $\lim_{x \to \infty} \ln x/x^p = 0$ for any positive power $p$ and $\lim_{x \to \infty} \sin x/x = 1$ (WebWorK 19).

Section 4.1 Maximum and Minimum Values

• Know the statement of the Extreme Value Theorem: If $f(x)$ is continuous on a closed interval $[a,b]$, then $f(x)$ has both an absolute min and an absolute max on $[a,b]$. (page 199, WebWorK 9).

• Know the definition of a critical number and how to find it: $x = c$ is a critical number of $f(x)$ if it is in the domain of $f$ and either $f'(c)=0$ or $f'(c)$ does not exist. (Example 5, WebWorK 1-4).

• Know the statement of Fermat's Theorem: If $f(c)$ is a local min or max, then $c$ is a critical number.

• Practice on using the three steps of the Closed Interval Method to find the extreme values of a continuous function $f(x)$ on a closed interval $[a,b]$ (page 202, Example 6, WebWorK 5-8)

Step 1: Find the critical numbers of $f(x)$ in $[a,b]$.
Step 2: Evaluate $f(x)$ at the critical numbers in $[a,b]$ and at the endpoints.
Step 3: The absolute min and max on $[a,b]$ are the smallest and largest among the values computed in Step 2.

Section 4.3: Derivatives and the Shapes of Graphs

• Know the Increasing/Decreasing Test and how to use it (Example 1):
  $f'(x)>0$ for $a<x<b$ $\Rightarrow$ $f$ is increasing on $(a,b)$
  $f'(x)<0$ for $a<x<b$ $\Rightarrow$ $f$ is decreasing on $(a,b)$

  The sign of $f'(x)$ can change only at the critical numbers. To find the sign of $f'(x)$ on the interval between two critical numbers, calculate the sign at any test point in that interval

• Know the First Derivative Test for finding local maxima or minima of a function (Examples 2 and 3, WebWorK 2):

<table>
<thead>
<tr>
<th>Sign change of $f'$ at $c$</th>
<th>Type of critical point</th>
</tr>
</thead>
<tbody>
<tr>
<td>From + to −</td>
<td>Local maximum</td>
</tr>
<tr>
<td>From − to +</td>
<td>Local minimum</td>
</tr>
</tbody>
</table>

If $f'$ does not change sign at $x = c$, then $f(c)$ is neither a local min nor max (unless $f$ is constant to the left or right of $c$).

• Be able to determine the behavior of a function $f$ given the graph of its derivative $f'$ (WebWorK 1).

• Know the Concavity Test:
  $f''>0$ $\Rightarrow$ $f$ is concave up
  $f''<0$ $\Rightarrow$ $f$ is concave down

• Know the definition of an inflection point (a point on the graph of $f$ where the concavity changes) and how to find it (if $f''(c)=0$ and $f''(x)$ changes sign at $c$, then $(c, f(c))$ is an inflection point). (WebWork 3, 4)

• Know the Second Derivative Test for finding local maxima or minima of a function:
  If $f''(c)=0$ and $f''(c)$ exists, then
  - $f(c)$ is a local maximum if $f''(c)<0$
  - $f(c)$ is a local minimum if $f''(c)>0$
  - The test fails if $f''(c)=0$.

• Given a function, be able to discuss it with respect to increasing/decreasing, concavity, points of inflection and local maxima and minima. Be able to sketch the curve based on this information. (Examples 5 and 6, WebWorK 5-10)
Section 4.4: Curve Sketching

- Read the Guidelines for Sketching a Curve on pages 220/221. For a given function you need to know how to determine: Domain, Intercepts, Symmetry, Asymptotes, Intervals of Increase or Decrease, Local Maximum and Minimum Values, Concavity and Points of Inflection. With all this information be able to sketch an accurate graph of the function. (Examples 1-4, WeBWorK 1-5).

Section 4.5 Optimization Problems

When solving Optimization Problems follow these guidelines:

1. Read the problem carefully. Ask yourself: What is the unknown? What are the given quantities? What are the given conditions? In most cases it is useful to draw a diagram to better understand the problem.

2. Introduce the variables: Assign a variable to the quantity that is to be maximized or minimized (let's call it Q) as well as all the other unknown quantities. Express Q in terms of the other variables.

3. If the quantity to be maximized or minimized, Q, is a function of more than one variable use the given information to find relationships (equations) among the variables so that you can express Q in terms of one variable only.

4. Use the methods of Sections 4.1 and 4.3 to find the absolute maximum or minimum value of the function Q.

Read Examples 1-5, WeBWorK 1-10.

Section 4.7 Antiderivatives

- Know the definition of antiderivative and memorize the Table of antiderivatives on page 243.(Examples 1 and 2, WeBWorK 1-10)

- Be able to determine the constant C when specific conditions on the function are given (see Examples 3 and 4, WeBWorK 11-19).

- Be able to work out applied problems about Rectilinear motion. In particular remember that the velocity is an antiderivative of the acceleration and the position function is an antiderivative of the velocity (see Examples 5, 6, WeBWorK 20-22)

Section 5.1 Areas and Distances

- Know how to approximate the area under a function by using the Left Sum, Right Sum and the Midpoint Sum (Example 1, WeBWorK 1).

- Understand how the exact area is defined by taking the limit as the number of rectangles, $n$, approaches infinity (Example 3).

- Understand the distance problem: given the velocity function (assume it is always positive), the distance traveled by an object during the time period from $t = a$ to $t = b$ is the area below the velocity function and this area can be approximated using rectangles (Example 4)

- Know how to approximate the area when the function is given by a formula (WeBWorK 1), a table (WeBWorK 2), or a graph.

Section 5.2 The Definite Integral

- Understand the definition of a definite integral as the limit of the Riemann sum (see Example 1, WeBWorK 3).

- Know how to approximate a definite integral using the Left Sum, the Right Sum or the Midpoint Sum (Example 4, WeBWorK 1.2)

- Understand the geometric interpretation of the definite integral as the signed area of the region between the graph of $f(x)$ and the $x$-axis (sum of the area above the $x$-axis minus the area below the $x$-axis) and be able to evaluate definite integrals by interpreting them in terms of areas (Example 3, WeBWorK 4-7).

- Know the properties of definite integrals (pages 269-271) and how to apply them (Examples 5, 6, 7, WeBWorK 8-10)

Section 5.3 Evaluating Definite Integrals

- Know how to evaluate definite integrals using the Evaluation Theorem (page 275). (Examples 1-2, 5, 6, WeBWorK 1-20)

- Know how to evaluate an indefinite integral and what it means (antiderivative) (Examples 3, 4, WeBWorK 21-33).

- Memorize the Table of Indefinite Integrals on page 277.

- Know how to work out the applications of The Net Change Theorem. Read all the examples on pages 279-281 (WeBWorK 35, 36) In particular, given the velocity function of a particle moving along a line, be able to determine the displacement and the distance traveled during a given time period. (see Formulas 2 and 3 on page 280 and Example 7, WeBWorK 34)
Section 5.4 The Fundamental Theorem of Calculus (will not be in Test 3 but it will be on the Final)

- Know the Fundamental Theorem of Calculus, Part I, and how to apply it (Examples 1, 2, 3,4, WeBWorK 2-4, 10,11).
  In particular make sure you practice on problems where you need to use the chain rule (if \( g(x) = \int_a^{h(x)} f(t) \, dt \) then
  \[ g'(x) = f(h(x)) \cdot h'(x) \] (Example 5 WeBWorK 5-9)
- Know how to work out problems where the graph of \( f(t) \) is given and you are asked to analyze the behavior of \( g(x) = \int_a^x f(t) \, dt \) (see Example 1 and WeBWorK 1). In this case recall that, since \( g'(x) = f(x) \) the function \( g(x) \) is increasing if \( f(x) \geq 0 \), that is, if the graph of \( f \) is above the \( x \)-axis. Also, since \( g''(x) = f''(x) \), the function \( g(x) \) is concave up if \( f''(x) \geq 0 \), that is, if the function \( f(x) \) is increasing.
- Understand that the Fundamental Theorem of Calculus, Part II is simply the Evaluation Theorem from section 5.3. Note that the FTC shows that integration and differentiation are inverse operations. By FTC I, if you start with a continuous function \( f(x) \) and form the integral \( \int_a^x f(t) \, dt \), then you get back the original function \( f(x) \) by differentiating:
  \[
  f(x) \rightarrow \int_a^x f(t) \, dt \rightarrow \frac{d}{dx} \int_a^x f(t) \, dt = f(x)
  \]
  On the other hand, by FTC II (or evaluation Theorem), if you first differentiate \( f(x) \) and then integrate, you also recover \( f(x) \) [but only up to a constant \( f(a) \)]:
  \[
  f(x) \rightarrow f'(x) \rightarrow \int_a^x f'(t) \, dt = f(x) - f(a)
  \]
- Know the definition of Average Value of a Function on an interval and how to find it (Example 6, WeBWorK 12-15)