Corollary \[ \text{Var}(X(s) \mid X(t)=b) = s \quad s > 0 \]

Interested in derivative of Brownian Wiener process

(Does not exist!)

Except Wiener process models displacement of Brownian particle, so velocity of Brownian particle should be derivative of Wiener process.

-So-

\[ X'(t) \text{ d.u.e.} \]

but: \[ X(t) = \int_0^t X'(t) \, dt \]

makes sense!

Consider equation

\[ y'(t) = ay(t) + bX(t) \]

in formulation

\[ y(t) - y(s) = a \int_s^t y(s) \, ds + b \int_s^t X(s) \, ds \]

- OR -

\[ y(t) = y(s) + a \int_s^t y(s) \, ds + bX(t) \]

Integral equation

Use variation of parameters formula

Guess \[ y(t) = y(s) e^{at} + b \int_s^t e^{a(t-s)} X(s) \, ds \]