Theorem (too hard) Brownian motion is continuous, i.e. \((-W(t, \omega) + W(t+\epsilon, \omega)) \to 0\) for (almost) each sample path \(\omega\).

Theorem Brownian motion is not differentiable for almost every sample path.

Proof (outline) Suppose it is differentiable on a set of positive probability, then

\[
\lim_{t \to 0} \frac{W(t) - W(0)}{t} = 0, \text{ i.e. } W(t) \text{ is } O(t)
\]

Need Lemma

\[E X(t) = 0\]

\[E X(t)^2 = t\]

\[E X(t)^q = \frac{3t^2}{2}\]

\[\text{Var}(X(t)^2) = \frac{t^2}{2}\]

So \(W(t)^2 = t + O(t^2)\)

So \(\frac{W(t)^2}{t} = 1 + O(t) - \text{ or } - W(t) = t^{\frac{1}{2}} + O(t^{\frac{3}{2}})\)

This is incompatible with \(W(t)\) is \(O(t)\)!