Sketch and find the area of the regions (note the ‘s’) bounded by

\[ y = x^2 \quad x = 2 \sin^2 y. \]

**Solution:**

Two regions can be seen from the graph.

![Graph of y = x^2 and x = 2 sin^2 y](image)

It is also clear from the graph that the curves intersect at \( x = y = 0 \). Since we cannot solve the equation \( \sqrt{y} = 2 \sin^2 y \) algebraically, we need to estimate the intersections using a graphing device. Additionally, this area will be easiest to find if we integrate with respect to \( y \). I found the next two points of intersection to be \((0.84, 0.705)\) and \((1.456, 2.12)\).

Using \( a = 0.705 \) and \( b = 2.12 \) the area of the regions will be

\[
\int_0^a (\sqrt{y} - 2 \sin^2 y) \, dy + \int_a^b (2 \sin^2 y - \sqrt{y}) \, dy
\]

The first integral yields

\[
\int_0^a (\sqrt{y} - 2 \sin^2 y) \, dy = \int_0^a \left( \sqrt{y} - 2 \cdot \frac{1 - \cos 2y}{2} \right) \, dy
\]

\[
= \int_0^a (\sqrt{y} - 1 + \cos 2y) \, dy
\]

\[
= \left( \frac{2}{3} y^{3/2} - y + \frac{1}{2} \sin 2y \right) \bigg|_0^a
\]

\[
= \frac{2}{3} a^{3/2} - a + \frac{1}{2} \sin 2a - 0 \approx 0.183
\]
From the second integral, we get
\[
\int_{a}^{b} \left( 2 \sin^2 y - \sqrt{y} \right) dy = \int_{a}^{b} \left( 2 \cdot \frac{1 - \cos 2y}{2} - \sqrt{y} \right) dy \\
= \int_{a}^{b} \left( 1 - \cos 2y - \sqrt{y} \right) dy \\
= \left[ y - \frac{1}{2} \sin 2y - \frac{2}{3} y^{3/2} \right]_{a}^{b} \\
= b - \frac{1}{2} \sin 2b - \frac{2}{3} b^{3/2} - a + \frac{1}{2} \sin 2a + \frac{2}{3} a^{3/2} \approx 0.691
\]

The area of the enclosed regions is \( \approx 0.183 + 0.691 = 0.874 \).