Classification of Differential Equations.

- **Ordinary and Partial**
  - Ordinary: regular derivatives...
  - Partial: has partial derivatives...

- **Order**: the highest derivative order in the d.e.
  - Ex: \( y' + 3xy = x^2 \) first order
  - \( y'' + y' - y = 0 \) second order

- **Linear**: can be written as:
  - \( a_n y^{(n)} + a_{n-1} y^{(n-1)} + \ldots + a_2 y'' + a_1 y' + a_0 y = b(x) \)
  - Nonlinear: \( y'' + y' = x^2 \) \( y'' + \sin(y) = x^4 \)

- **Homogeneous**: the \( b(x) \) function is set = 0.
  - \( b(x) \) is called "the forcing function".
  - Ex: \( y'' = 2y \rightarrow y'' - 2y = 0 \) it is homogeneous.
  - In a linear d.e., if there are any terms that do not contain \( y \) (or its derivs), then it is not homogeneous.
    - \( y'' + 3xy' + 2x = 0 \rightarrow y'' + 3xy' = -2x \) not homogeneous.

- **Autonomous** (or autonomic): the d.e. does not contain the input variable anywhere.
  - \( y' = y \)
  - \( y'' + 3y' + 4y = 0 \)
Basic Solution Techniques:

\[ y' = y \Rightarrow y = e^x \Rightarrow y = Ce^x \]

where \( C \) is any constant.

"The rate of change of a population is proportional to that population."

\[ \Rightarrow p'(t) = kp(t) \]

Let \( p(t) = \text{pop at time } t, \) so "rate of change of population" is \( p'(t). \)

\[ y' = 3y \Rightarrow y = e^{3x} \text{ check: } y' = 3e^{3x} \]

\[ 3e^{3x} = 3(e^{3x}) \text{ } \checkmark \text{ true!} \]

General Solution: \( y = Ce^{3x} \)

\[ y' = ky \Rightarrow y = Ce^{kt} \text{ is the general solution.} \]

**Ex.** Suppose the rate of change of a population is one-tenth of the population. The population after 2 years is 1000. Find \( p(t) \), the population after \( t \) years.

\[ p'(t) = 0.1P(t) \]

\[ \Rightarrow p' = 0.1p \]

General Solution: \( p(t) = Ce^{0.1t} \)

After 2 yrs, \( P = 1000: \)

\[ 1000 = Ce^{0.1(2)} \]

\[ \Rightarrow 1000 = Ce^{0.2} \Rightarrow C = \frac{1000}{e^{0.2}} \approx 818. \]

Specific Sol: \( p(t) = 818e^{0.1t} \)
The rate of change of a population of bunny rabbits is 0.06 of the population. After 6 months, there are 300 bunny rabbits. Find \( P(t) \), the population after \( t \) months. Then find the initial population. Then find the population after 2 yrs.

**Solution:**

\[ P' = 0.06P \quad (6, 300) \]

\[ \Rightarrow P(t) = Ce^{0.06t} \quad \text{(General Solution)} \]

\[ 300 = Ce^{0.06(6)} \Rightarrow C = \frac{300}{e^{0.36}} \approx 209 \]

**Specific Sol.:**

\[ P(t) = 209e^{0.06t} \]

Initial population is when \( t = 0 \):

\[ P(0) = 209. \]

After 2 years:

\[ P(24) = 209e^{0.06(24)} \approx 884 \]

Another way to solve \( y' = y \)

Write \( y' \) as \( \frac{dy}{dx} \), so we have \( \frac{dy}{y} = dx \)

\[ \Rightarrow \int \frac{dy}{y} = \int dx \]

(Integrate both sides: \( \ln|y| = x + C \)

(Separation of variables)

Base-e both sides:

\[ |y| = e^{x+C} \]

\[ |y| = e^xe^{C} \]

\[ |y| = Ce^{x} \] when \( C = e^{C} \)

\[ y = Ce^{x} \]