1. Evaluate

- **Form A:** $\int_0^3 \int_{\sqrt{9-x^2}}^{2x^2 + 2y^2} (2x^2 + 2y^2) \, dy \, dx = \int_{-\pi/2}^{\pi/2} \int_0^3 2r^2 \, r \, dr \, d\theta = 2 \int_{-\pi/2}^{\pi/2} \int_0^3 r^3 \, dr \, d\theta = 2 \int_{-\pi/2}^{\pi/2} \left[ \frac{1}{4} r^4 \right]_0^3 \, d\theta = \frac{2(81)}{4} \int_{-\pi/2}^{\pi/2} \, d\theta = \frac{1}{2} (81)(\pi) = \frac{81}{2} \pi.$

- **Form B:** $\int_{-4}^{0} \int_{\sqrt{16-x^2}}^{(3x^2 + 3y^2)} dx \, dy = \int_{\pi/2}^{\pi} \int_0^4 3r^2 \, r \, dr \, d\theta = 3 \int_{\pi/2}^{\pi} \int_0^4 r^3 \, dr \, d\theta = 3 \int_{\pi/2}^{\pi} \left[ \frac{1}{4} r^4 \right]^4_0 \, d\theta = 192 \int_{\pi/2}^{\pi} \, d\theta = 192 \left( \frac{\pi}{2} \right) = 96\pi.$

2. Set up a triple integral over region \( S \) in \( R^3 \), where \( S \) is bounded by the \( xz \)-plane, the \( yz \)-plane, the plane \( x + 4y - z = 0 \) and the plane \( z = 6 \). Just set it up, do not solve. Use \( f(x, y) \) as the integrand. A picture is below.

There are many correct answers. Some are:

**Form A:** $\int_0^6 \int_0^3 \int_{\frac{1}{3}x^3}^{x+4y} f(x, y, z) \, dz \, dy \, dx + \int_0^3 \int_0^6 \int_{\frac{1}{3}x^3}^{x+4y} f(x, y, z) \, dz \, dx \, dy + \int_0^6 \int_0^4 \int_{\frac{1}{3}x^3}^{x+4y} f(x, y, z) \, dz \, dx \, dy$

**Form B:** $\int_0^9 \int_{\frac{1}{3}x^3}^{x+3y} \int_{\frac{1}{3}x^3}^{x+3y} f(x, y, z) \, dz \, dy \, dx + \int_0^3 \int_{\frac{1}{3}x^3}^{x+3y} \int_{\frac{1}{3}x^3}^{x+3y} f(x, y, z) \, dz \, dx \, dy + \int_0^6 \int_{\frac{1}{3}x^3}^{x+3y} \int_{\frac{1}{3}x^3}^{x+3y} f(x, y, z) \, dz \, dx \, dy$

3. Let \( S \) be the solid above the \( xy \)-plane bounded by the spheres \( x^2 + y^2 + z^2 = 4 \) and \( x^2 + y^2 + z^2 = 9 \). Evaluate

\[ \iiint_S \left( 1 + x^2 + y^2 + z^2 \right) \, dV. \]

**Form A:** $\int_0^{\pi/2} \int_0^{2\pi} \int_0^3 (1 + \rho^2) \rho^2 \sin \phi \, d\rho \, d\theta \, d\phi = \int_0^{\pi/2} \int_0^{2\pi} \int_0^3 (\rho^4 + \rho^2) \sin \phi \, d\rho \, d\theta \, d\phi.$

With respect to \( \rho \): $\int_0^3 (\rho^4 + \rho^2) \, d\rho = \left[ \frac{1}{3} \rho^3 + \frac{1}{5} \rho^5 \right]^3_0 = \left( \frac{9}{3} + \frac{243}{5} \right) - \left( \frac{8}{3} + \frac{32}{5} \right) = \frac{728}{15}.$

With respect to \( \theta \): $\frac{728}{15} \frac{4\pi}{\pi} = \frac{1456}{15} \pi.$

With respect to \( \phi \): $\frac{1456}{15} \pi \int_0^{\pi/2} \sin \phi \, d\phi = \frac{1456}{15} \pi (-\cos \phi)^{\pi/2} = \frac{1456}{15} \pi (0 - (-1)) = \frac{1456}{15} \pi.$

**Form B:** $\int_0^{\pi/2} \int_0^{\pi/2} \int_1^2 (1 + \rho^2) \rho^2 \sin \phi \, d\rho \, d\theta \, d\phi + \int_0^{\pi/2} \int_0^{\pi/2} \int_1^2 (\rho^2 + \rho^4) \, d\phi \, d\theta \, d\phi.$

With respect to \( \rho \): $\int_1^2 (\rho^2 + \rho^4) \, d\rho = \left[ \frac{1}{3} \rho^3 + \frac{1}{5} \rho^5 \right]^2_1 = \left( \frac{8}{3} + \frac{32}{5} \right) - \left( \frac{1}{3} + \frac{1}{5} \right) = \frac{128}{15}.$

With respect to \( \theta \): $\frac{128}{15} \frac{4\pi}{\pi} = \frac{64}{15} \pi.$

With respect to \( \phi \): $\frac{64}{15} \pi \int_0^{\pi/2} \sin \phi \, d\phi = \frac{64}{15} \pi (-\cos \phi)^{\pi/2} = \frac{64}{15} \pi (0 - (-1)) = \frac{64}{15} \pi.$
4. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x,y) = (x + y, 2y)$ and $C$ is the path $y = \sqrt{x}$ from $(1,1)$ to $(9,3)$.

- Parameterize: $\mathbf{r}(t) = (t, \sqrt{t})$, $1 \leq t \leq 9$
- Thus, $\mathbf{r}'(t) = (1, \frac{1}{2\sqrt{t}})$ and $\mathbf{F}(t) = (t + \sqrt{t}, 2\sqrt{t})$.
- The dot product is $t + \sqrt{t} + 1$.
- Thus, $\int_C \mathbf{F} \cdot d\mathbf{r} = \int_1^9 (t + \sqrt{t} + 1) \, dt = \left[ \frac{1}{2} t^2 + \frac{2}{3} t^{3/2} + t \right]_1^9 = \left( \frac{81}{2} + 18 + 9 \right) - \left( \frac{1}{2} + \frac{2}{3} + 1 \right) = \frac{196}{3}$.

5. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x,y) = (3x^2y + y, x^3 + x)$ and $C$ is a line segment from $(2,1)$ to $(4,3)$, then to $(3, -1)$.

**Form A:**
- Check to see if $\mathbf{F}$ is conservative: $M_y = 3x^2 + 1, N_x = 3x^2 + 1$. They are equal, so $\mathbf{F}$ is conservative.
- The potential function is $f(x,y) = x^3y + xy$.
- Thus, $\int_C \mathbf{F} \cdot d\mathbf{r} = [x^3y + xy]_{(2,1)}^{(3,-1)} = (-27 - 3) - (8 + 2) = -40$.

**Form B:** The only difference was that $f(x,y) = x^3y + y$, so the value of the line integral is $-7$.

6. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x,y) = (14x + 10y, 13x - 11y)$ and $C$ is a sequence of line segments from $(0,0)$ to $(10,0)$ to $(6,4)$ to $(0,0)$.

**Form A:**
- Check to see if $\mathbf{F}$ is conservative: $M_y = 10, N_x = 13$. They are not equal, so $\mathbf{F}$ is not conservative.
- The path is a closed simple loop in the counter-clockwise direction. The interior is a triangle with base 10 and height 4.
- Using Green’s Theorem, we have $\int_C \mathbf{F} \cdot d\mathbf{r} = \iint_D 3 \, dA = 3(\text{area of } D) = \frac{3}{2} (10)(4) = 60$.

**Form B:** Same problem except the triangle had base 10 and height 6, thus the answer was 90.

7. Suppose $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x,y) = (e^x, \sin(y))$ is evaluated along a right triangle in the counter-clockwise manner such that the line integral along one leg is 6, and the line integral along the other leg is $-2$. What is the value of the line integral evaluated along the hypotenuse?

Since $\mathbf{F}$ is conservative and the path is a loop, the line integral is 0, so the sum of the three sides must be 0. Thus the line integral along the hypotenuse is $-4$. 
8. A particle moves from B to A along the bold-face path within the vector field shown below. Is the net work done by the vector field on this particle positive, negative, or zero?

From A to B: negative; From B to A: positive.

9. True or false: the work line integral along any path in a conservative vector field is always 0.

“along any path” false; “along any loop path” true.

10. Below is a solid bounded below by a cone and above by a sphere. The point indicated lies on the rim. Give the bounds of this solid in spherical coordinates.

\[ 0 \leq \rho \leq \sqrt{21}; \quad 0 \leq \theta \leq 2\pi; \quad 0 \leq \phi \leq \cos^{-1}\left(\frac{4}{\sqrt{21}}\right). \]

It was possible to use arctan or arcsin in the final part of the bound for phi.