1. What is the domain of \( f(x, y) = \frac{1}{\sqrt{2x-y}} \)?
2. What is the domain of \( g(x, y) = \ln(x^2 + 4y) \)?
3. Find all first and second-order partial derivatives of \( f(x, y, z) = x^2 y^3 z^5 \).
4. Find the slope of the tangent line to \( f(x, y) = x^3 y - y^2 \) when \( x = 1 \) and \( y = 2 \), in the direction of \( x = 4 \), \( y = 3 \).
5. Find the direction of the steepest slope of \( g(x, y) = \frac{x}{y^2} \) at \( x = 3 \), \( y = -2 \). Then find the slope.
6. Find a vector normal to the surface \( h(x, y) = 3xy + y^2 - x^3 \) at \( x = 4 \), \( y = 2 \).
7. Find the equation of the tangent plane in #6.
8. Larry is measuring a circular cylinder. He measures the height to be 8 m, with a tolerance of \( \pm 2 \) cm, and the radius as 3 m, with a tolerance of \( \pm 3 \) cm. Using differentials, find the approximate range of tolerance in the volume of this solid.
9. Let \( g(x, y) = x^3 + y^3 - 3x - 12y + 1 \). Find all critical points and classify them as min, max or saddle points.
10. Consider the map below. Assume C is a saddle, D is a minimum and A is a maximum, and that the surface is everywhere differentiable.

\[ \begin{align*}
\frac{\partial f}{\partial x}(A) &= & \frac{\partial f}{\partial x}(B) &= & \frac{\partial f}{\partial x}(E) &= \\
\frac{\partial f}{\partial y}(A) &= & \frac{\partial f}{\partial y}(B) &= & \frac{\partial f}{\partial y}(E) &= 
\end{align*} \]

b) Suppose \( P \) is a path (constraint). Approximate the minimum and maximum values of \( f \) constrained to path \( P \).

c) On the map, draw in the gradient vector at E.
11. Let \( f(x, y) = 3 \sin(x^2 - 4y) \). Find \( \nabla f \).

12. Suppose \( y = f(x(s, t), y(s, t)) \), and suppose that \( \frac{\partial f}{\partial t} = 10, \frac{\partial f}{\partial x} = 2, \frac{\partial f}{\partial y} = 1, \frac{\partial x}{\partial s} = -3, \frac{\partial x}{\partial t} = 4 \) and \( \frac{\partial y}{\partial s} = 5 \). Find the value of \( \frac{\partial y}{\partial t} \).

13. Find the volume below \( z = 2e^{x^2+y^2} \) over the region in the \( xy \)-plane shown below.

14. Let \( z = f(x, y) = 2x^2 + xy - \frac{1}{4}y^4 + 4x - 2y + 1 \). Find the equation of the tangent plane at \( x_0 = -2 \) and \( y_0 = 3 \), then use it to estimate \( f(-2.05, 3.1) \). Do not use a calculator.

15. Find \( \int_{-1}^{1} \int_{3}^{4} (x - 1) \, dy \, dx \).

16. Rewrite as a single double-integral: \( \int_{0}^{2} \int_{x}^{2x} xy \, dy \, dx + \int_{2}^{6} \int_{0}^{6-x} xy \, dy \, dx \).

17. Using only geometry, find the volume below \( f(x, y) = \sqrt{1 - x^2 - y^2} \) over the region in \#13.

18. Find the volume contained below the paraboloid \( z = 4 - x^2 - y^2 \) and above the \( xy \)-plane.

19. Reverse the order of integration: \( \int_{0}^{3} \int_{\sqrt{9-y}}^{\sqrt{9-y}} dx \, dy \).

20. Evaluate \#19.

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**Answers.** (report errors to surgent@asu.edu)

1. \( \{(x, y) \mid y < 2x \} \)
2. \( \{(x, y) \mid y > -\frac{1}{4}x^2 \} \)
3. \( f_x = 2xy^3z^5, f_y = 3x^2y^2z^5, f_z = 5x^2y^3z^4, \)
   \( f_{xx} = 2y^3z^5, f_{xy} = 6xy^2z^5, f_{xz} = 10xy^3z^4 \)
   \( f_{yx} = 6xy^2z^5, f_{yy} = 6x^2yz^5, f_{yz} = 15x^2y^2z^4 \)
   \( f_{zx} = 10xy^3z^4, f_{zy} = 15x^2y^2z^4, f_{zz} = 20x^2y^3z^3 \)
4. Direction is \( v = (3,1) \) so we need the unit direction vector, \( \left( \frac{3}{\sqrt{10}}, \frac{1}{\sqrt{10}} \right) \). The gradient of \( f \) is \( \nabla f = (f_x, f_y) = (3x^2y, x^3 - 2y) \). Evaluated at \( (1,2) \), we have \( \nabla f(1,2) = (6, -3) \). Thus, the slope will be \( \left( \frac{3}{\sqrt{10}}, \frac{1}{\sqrt{10}} \right) \cdot (6, -3) = \frac{18}{\sqrt{10}} - \frac{3}{\sqrt{10}} = \frac{15}{\sqrt{10}} \approx 4.743 \).
5. The direction of the steepest slope is the gradient of \( g \) evaluated at the given point. Thus, \( \nabla g = \left( \frac{1}{\sqrt{2}}, -\frac{2x}{y^3} \right) \). The slope is \( \left| \left( \frac{1}{\sqrt{2}}, \frac{3}{4} \right) \right| = \left( \frac{1}{\sqrt{2}} \right)^2 + \left( \frac{3}{4} \right)^2 = \frac{10}{4} \approx 0.791 \).
6. Gradient vectors are always orthogonal to a contour line, so we create a 4-dimensional function \( H(x, y, z) = 3xy + y^2 - x^3 - z \). Note that \( z = 3xy + y^2 - x^3 \) is a contour of \( H \), so that \( \nabla H = (3y - 3x^2, 3x + 2y, -1) \), and at \( (4,2) \), we have \( \nabla H(4,2) = (-42, 16, -1) \). Also acceptable is \( (42, -16, 1) \).
7. The point is \( (4,2, -36) \) so we have \(-42(x - 4) + 16(y - 2) - (z + 36) = 0 \).
8. The volume is \( V(r, h) = \pi r^2 h \), so therefore, \( dV = 2\pi rh \, dr + \pi r^2 \, dh \). Evaluating, we have \( dV = 2\pi(3)(8)(0.03) + \pi(3)^2(0.02) = \pm 5.089 \) cubic meters.
9. (1,2,-17) min, (-1,2,-13) saddle, (1,-2,15) saddle, (-1,-2,19) max.

10. a) 
\[ f_x(A) = 0 \quad f_x(B) = + \quad f_x(E) = + \]
\[ f_y(A) = 0 \quad f_y(B) = - \quad f_y(E) = - \]

b) Min, about 45, max about 115

c) Orthogonal to the contour, positive slope.

11. \( \nabla f = (6x \cos(x^2 - 4y) , -12 \cos(x^2 - 4y)) \).

12. \[ \frac{\partial f}{\partial t} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial f}{\partial y} \frac{\partial y}{\partial t} \rightarrow 10 = (2)(4) + (1) \frac{\partial y}{\partial t} \rightarrow \frac{\partial y}{\partial t} = 10 - 8 = 2 \). Here, y and f are interchangeable.

13. Use polar bounds. The bounds are \( 0 \leq r \leq 1, 0 \leq \theta \leq \frac{\pi}{2} \).

\[ \int_{0}^{\pi/2} \int_{0}^{1} 2e^{r^2} r \, dr \, d\theta = \int_{0}^{\pi/2} \left. \left[ e^{r^2} \right] \right|_{0}^{1} d\theta = \int_{0}^{\pi/2} (e - 1) \, d\theta = (e - 1) \int_{0}^{\pi/2} d\theta = \frac{(e - 1)\pi}{2}. \]

14. \( f_x(x, y) = 4x + y + 4 \rightarrow f_x(-2,3) = -1 \), and \( f_y(x, y) = x - y^3 - 2 \rightarrow f_y(-2,3) = -31 \). We also have \( z_0 = -31.25 \). Thus, the equation of the tangent plane is \( z + 31.25 = -(x + 2) - 31(y - 3) \). Thus,

\[
\begin{align*}
z + 31.25 &= -(-2.05 + 2) - 31(3.1 - 3) \\
z + 31.25 &= -(-0.05) - 31(0.1) \\
z + 31.25 &= 0.05 - 3.1 \\
z + 31.25 &= -3.05 \\
z &= -3.05 - 31.25 \\
z &= -34.3
\end{align*}
\]

15. Do the inside integral first: \( \int_{3}^{4} x(1 - y) \, dy = (xy - \frac{1}{2}xy^2)_{3}^{4} = (4x - 8x) - \left( 3x - \frac{9}{2}x \right) = -\frac{5}{2}x \).

Now integrate with respect to x: \( \int_{-1}^{2} \left( \frac{-5}{2}x \right) \, dx = \left( \frac{-5}{4}x^2 \right)_{-1}^{2} = \left( -\frac{5}{4} \cdot 4 \right) - \left( -\frac{5}{4} \cdot 1 \right) = -\frac{15}{4} \).

16. \( \int_{0}^{\pi/2} \int_{y/2}^{y} xy \, dx \, dy \)

17. The solid is a hemisphere of radius 1, but it’s being integrated over a quarter circle, so we have just 1/8 of a sphere. The volume of a sphere of radius 1 is \( \frac{4}{3}\pi(1)^3 = \frac{4}{3}\pi \), then divided by 8, we have \( \frac{1}{6}\pi \).

18. The region of integration is a circle of radius 2, so it’s advised to use polar. Thus, \( r \) ranges between 0 and 2, \( \theta \) between 0 and \( 2\pi \). The integrand is \( (4 - r^2)r \), where the extra \( r \) is from the Jacobian. Thus, we have \( \int_{0}^{2\pi} \int_{0}^{2} (4r - r^3) \, dr \, d\theta \). The inner integral works out to be 4, the out to be \( 2\pi \), so the volume is \( 8\pi \).

19. The region is the parabola \( y = 9 - x^2 \) above the \( x \)-axis, so we have \( \int_{-3}^{3} \int_{0}^{9-x^2} dy \, dx \).

20. 36.