Line Integrals Practice

No answers are provided. If you want to discuss them, go to Piazza and post your work there for feedback. I want to encourage such discussions on Piazza.

You are given a vector field \( \mathbf{F}(x, y) = (M(x, y), N(x, y)) \) and a path in the \( xy \)-plane \( r(t) = (x(t), y(t)) \), where \( a \leq t \leq b \). The following integrals,

\[
\int \mathbf{F} \cdot \mathbf{T} \, ds \quad \int \mathbf{F} \cdot d\mathbf{r} \quad \int M \, dx + N \, dy
\]

are all equivalent. They all describe a work-line integral.

Hints: Check to see if the vector field is a gradient field … consider using Green’s Theorem.

1. Find \( \int \mathbf{F} \cdot \mathbf{T} \, ds \), where \( \mathbf{F}(x, y) = (3x, 2xy) \) and \( C \) is a line segment from \((1,2)\) to \((4,7)\).
2. Find \( \int \mathbf{F} \cdot d\mathbf{r} \), where \( \mathbf{F}(x, y) = (xy^2, 2x - y) \) and \( C \) is a line segment from \((6,6)\) to \((3,0)\).
3. Find \( \int M \, dx + N \, dy \), where \( \mathbf{F}(x, y) = (x^2 + 2y, -x) \) and \( C \) is a line segment from \((-3,3)\) to \((5,1)\).
4. Find \( \int \mathbf{F} \cdot \mathbf{T} \, ds \), where \( \mathbf{F}(x, y) = (6y, x^2) \) and \( C \) is the parabola \( y = x^2 \) from \((1,1)\) to \((4,16)\).
5. Find \( \int M \, dx + N \, dy \), where \( \mathbf{F}(x, y) = (4x^2 + y) \) and \( C \) is the cubic \( y = x^3 \) from \((-1,-1)\) to \((2,8)\).
6. Find \( \int \mathbf{F} \cdot d\mathbf{r} \), where \( \mathbf{F}(x, y) = (x^3, 2) \) and \( C \) is the curve \( y = 1 - x^2 \) from \((0,1)\) to \((2,-3)\).
7. Find \( \int \mathbf{F} \cdot d\mathbf{r} \), where \( \mathbf{F}(x, y) = (3 - x, 1 + y) \) and \( C \) is a circle of radius 2, centered at the origin, traversed counterclockwise.
8. Find \( \int M \, dx + N \, dy \), where \( \mathbf{F}(x, y) = (2xy, 1 + 3x) \) and \( C \) is a line from \((1,2)\) to \((5,1)\), then to \((4,4)\).
9. Find \( \int \mathbf{F} \cdot \mathbf{T} \, ds \), where \( \mathbf{F}(x, y) = (8xy, 4x^2) \) and \( C \) is a sequence of line segments from \((0,0)\) to \((1,3)\) to \((4,7)\) to \((2,1)\).
10. Find \( \int \mathbf{F} \cdot d\mathbf{r} \), where \( \mathbf{F}(x, y) = (2x, 3y^2) \) and \( C \) is a quarter circle of radius 4 centered at the origin from \((4,0)\) to \((0,4)\).
11. Find \( \int M \, dx + N \, dy \), where \( \mathbf{F}(x, y) = (y + 2xy^3, x + 3x^2y^2) \) and \( C \) is the curve \( y = x^3 + x^2 - 2x + 1 \) from \((0,1)\) to \((1,1)\).
12. Find \( \int \mathbf{F} \cdot \mathbf{T} \, ds \), where \( \mathbf{F}(x, y) = (\cos y, -x \sin y) \) and \( C \) is a circle of radius 7, centered at \((5,1)\).
13. Find \( \int \mathbf{F} \cdot d\mathbf{r} \), where \( \mathbf{F}(x, y) = (2x, 4xy) \) and \( C \) is a triangle from \((0,0)\) to \((5,0)\) to \((5,5)\) back to \((0,0)\).
14. Find \( \int M \, dx + N \, dy \), where \( \mathbf{F}(x, y) = (100y, 101x) \) and \( C \) is a circle of radius 1, centered at \((\pi, \sqrt{2})\).
15. Find \( \int \mathbf{F} \cdot \mathbf{T} \, ds \), where \( \mathbf{F}(x, y) = (2x, 80y^{10} + 3x) \) and \( C \) is the parabola \( y = 1 - x^2 \) traversed from \((1,0)\) to \((-1,0)\), then back to \((1,0)\) along the x-axis.