Calculus-III Tangent Planes Practice Problems.

Answers are not included. You are encouraged to work together and post ideas and comments on Piazza.

**Example:** Find the equation of the tangent plane to \( f(x,y) = x^4y - y^2 \) at \( x_0 = 2 \) and \( y_0 = 3 \), and use the tangent plane to estimate the value of \( f(2.1,2.95) \).

**Solution:** We need the full point: \( z_0 = f(x_0,y_0) = (2)^4(3) - (3)^2 = 39 \). Thus, the point of tangency is \( (2,3,39) \).

The general form of a tangent plane is \( z - z_0 = f_x(x_0,y_0)(x - x_0) + f_y(x_0,y_0)(y - y_0) \).

Find the partial derivatives and evaluate at \( x_0 = 2 \) and \( y_0 = 3 \):

\[
\begin{align*}
    f_x(x,y) &= 4x^3y & f_x(2,3) &= 4(2)^3(3) = 96 \\
    f_y(x,y) &= x^4 - 2y & f_y(2,3) &= (2)^4 - 2(3) = 10
\end{align*}
\]

Now, assemble your plane:

\[
z - 39 = 96(x - 2) + 10(y - 3).
\]

Simplify by clearing parentheses: \( z - 39 = 96x - 192 + 10y - 30 \), then isolate \( z \):

\[
z = 96x + 10y - 183.
\]

To estimate \( f(2.1,2.95) \), use the plane:

\[
z = 96(2.1) + 10(2.95) - 183 \\
= 201.6 + 29.5 - 183 \\
= 48.1.
\]

The **actual** value of \( f(2.1,2.95) \) is \( (2.1)^4(2.95) - (2.95)^2 = 48.669395 \ldots \). Note that the plane gave a very close approximation that uses easier arithmetic. This is also a good check of your work.

1. Find the equation of the tangent plane to \( f(x,y) = x^2 + y^2 \) at \( x_0 = 3 \) and \( y_0 = 4 \), and use the tangent plane to estimate the value of \( f(3.1,3.9) \).
2. Find the equation of the tangent plane to \( f(x,y) = 2xy^2 \) at \( x_0 = 1 \) and \( y_0 = -2 \), and use the tangent plane to estimate the value of \( f(1.05,-1.9) \).
3. Find the equation of the tangent plane to \( f(x,y) = x^3y - 2x \) at \( x_0 = -1 \) and \( y_0 = 3 \), and use the tangent plane to estimate the value of \( f(-1.02,3.04) \).
4. Find the equation of the tangent plane to \( f(x,y) = \sqrt{x - 2y} \) at \( x_0 = 5 \) and \( y_0 = 1 \), and use the tangent plane to estimate the value of \( f(5.1,1.06) \).
5. Find the equation of the tangent plane to \( f(x,y) = \ln(x^2 - y^3) \) at \( x_0 = 4 \) and \( y_0 = 2 \), and use the tangent plane to estimate the value of \( f(4.2,1.9) \).
6. Find the equation of the tangent plane to \( f(x,y) = \frac{6x}{y} \) at \( x_0 = 1 \) and \( y_0 = 4 \), and use the tangent plane to estimate the value of \( f(1.04,3.98) \).
7. Find the equation of the tangent plane to \( f(x, y) = \frac{x+y}{x-y} \) at \( x_0 = -2 \) and \( y_0 = 5 \), and use the tangent plane to estimate the value of \( f(-2, 1, 4.9) \).

8. Find the equation of the tangent plane to \( f(x, y) = e^{xy} \) at \( x_0 = 1 \) and \( y_0 = 2 \), and use the tangent plane to estimate the value of \( f(1, 1, 93) \).

9. Find the equation of the tangent plane to \( f(x, y) = x \sin y \) at \( x_0 = 6 \) and \( y_0 = \frac{\pi}{2} \), and use the tangent plane to estimate the value of \( f(6, 1, 1.5) \).

10. Find the equation of the tangent plane to \( f(x, y) = x^3 - y^2 \) at \( x_0 = 8 \) and \( y_0 = 11 \), and use the tangent plane to estimate the value of \( f(8, 2, 11.2) \).