Bounds Practice – Polar, Cylindrical and Spherical

Describe these regions using polar bounds.

1. 

2. 

3. 

4. 

Translate these integrals into polar coordinates. Then find their exact answers. Hint: draw the regions.

5. \[ \int_0^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \sqrt{x^2 + y^2} \, dy \, dx \]

6. \[ \int_{-3}^3 \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} (1 + 2x^2 + 2y^2) \, dy \, dx \]

7. \[ \int_0^3 \int_{\sqrt{9-x^2}}^{\sqrt{25-x^2}} \frac{1}{4x^2 + 4y^2 + 1} \, dy \, dx + \int_3^5 \int_0^{\sqrt{25-x^2}} \frac{1}{4x^2 + 4y^2 + 1} \, dy \, dx \]
9. Find the volume contained within the paraboloid \( z = 16 - x^2 - y^2 \) that lies above the xy-plane.

10. Find the volume within the paraboloid \( z = 25 - x^2 - y^2 \) that lies in the first octant.

11. Find the volume within the paraboloid \( z = 20 - x^2 - y^2 \) that lies above the plane \( z = 11 \).

12. Find the volume of the paraboloid \( z = 20 - x^2 - y^2 \) that lies above the xy-plane but below the plane \( z = 11 \).

13. Find the volume within the paraboloids \( z = x^2 + y^2 \) and \( z = 18 - x^2 - y^2 \).

14. Find the volume within the cylinder \( x^2 + y^2 = 36 \) that lies above the xy-plane and below \( x + z = 12 \).

Evaluate these integrals.

15. \( \iiint_R x^2 + y^2 + z^2 \, dV \), where \( R \) is a sphere centered at the origin of radius 3.

16. \( \iiint_R \sqrt{x^2 + y^2 + z^2} \, dV \), where \( R \) is a hemisphere above the xy-plane with radius 2.

17. \( \iiint_R (2x^2 + 2y^2 + 2z^2 + 1) \, dV \), where \( R \) is the space between spheres centered at the origin of radii 2 and 4, within the first octant.

18. \( \int_0^4 \int_{\sqrt{16-x^2}}^{\sqrt{16-x^2}} \int_0^{\sqrt{16-x^2-y^2}} x \, dz \, dy \, dx \).

19. A hemisphere of radius 6 centered at the origin and above the xy-plane has a conical sector removed such that the point (2,1,4) lies on the rim. Find the volume of the remaining solid.

20. Two spheres of radius 1 intersect one another such that each sphere passes through the other’s center, creating a lens-shaped region. Find that region’s volume.