Line Integral Practice

**Scalar Form.** The general form is \( \int_C f(x, y) \, ds = \int_a^b f(x(t), y(t))|r'(t)| \, dt \), where the surface \( S \) is given as \( z = f(x, y) \), and the path \( C \) over which the integral is evaluated is defined by the vector-valued function \( r(t) = (x(t), y(t)) \).

The usual method of solving a scalar line integral is to (1) parameterize the path in terms of variable \( t \). This gives you your \( x \) and \( y \) as functions of \( t \). Be sure to note the bounds of \( t \), which gives you the bounds of integration; (2) Find \( |r'(t)| \); (3) Replace \( x \) and \( y \) in the integrand with their equivalents in terms of \( t \), and replace \( ds \) with \( |r'(t)| \, dt \); (4) Simplify and evaluate. Use a calculator if necessary.

Set up and solve:

1. Evaluate \( \int_C f(x, y) \, ds \), where \( z = x^2 + 3y \) over the path \( C \) which is the straight line from (1,2) to (3,1).
2. Evaluate \( \int_C f(x, y) \, ds \), where \( z = xy^2 \) over the path \( C \) which is the portion of the cubic \( y = x^3 \) from (0,0) to (2,8).
3. Evaluate \( \int_C f(x, y) \, ds \), where \( z = 2x + y^2 \) over the path \( C \) which is the portion of a circle of radius 1, centered at the origin, starting at (1,0) and ending at (0,1).

Answers: 1. \( \frac{53}{6} \); 2. 308.969 (used a calculator); 3. \( 2 + \frac{\pi}{4} \).

**Vector (Circulation) Form.** The general form is \( \int_C \mathbf{F} \cdot \mathbf{T} \, ds = \int_C \mathbf{F} \cdot d\mathbf{r} = \int_a^b P(x, y)dx + Q(x, y)dy \). You’ll be given a vector field \( \mathbf{F}(x, y) = \langle P(x, y), Q(x, y) \rangle \) and a path \( C = r(t) = \langle x(t), y(t) \rangle \).

To solve, you parameterize the path in terms of \( t \), which gives you the functions \( x(t) \) and \( y(t) \). Replace the \( x \) and \( y \) in the functions \( P \) and \( Q \) with the new versions in terms of \( t \). Then simplify and evaluate. All line integrals are in terms of \( t \). Note that the three vector line integral forms shown above all mean the same thing. Get used to seeing these and be able to recognize it as a vector line integral when you see it.

Set up and solve:

4. Evaluate \( \int_C \mathbf{F} \cdot \mathbf{T} \, ds \) where \( \mathbf{F}(x, y) = \langle x^2, x + y \rangle \) and \( C \) is the straight line from (4,0) to (2,2).
5. Evaluate \( \int_C \mathbf{F} \cdot d\mathbf{r} \) where \( \mathbf{F}(x, y) = \langle 3x - 2y, y^2 \rangle \) and \( C \) is the portion of a parabola \( y = x^2 \) from (1,1) to (3,9).
6. Evaluate \( \int_a^b P(x, y)dx + Q(x, y)dy \) where \( \mathbf{F}(x, y) = \langle x^2, 1 - y \rangle \) and \( C \) is a circle of radius 2 centered at the origin starting from (2,0) and traveling counterclockwise to (-2,0).

Answers: 4. \( -\frac{32}{2} \); 5. \( \frac{712}{3} \); 6. \( \frac{16}{3} \).

Report any errors to surgent@asu.edu.