Graph \( y = x^2 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>4</td>
</tr>
<tr>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
</tbody>
</table>

\( (x, y) \) how do we know these connect with a curve? Plot more pts, but the same question persists.

\[ \text{to answer such a question, we have to use calculus.} \]

"At \( x \) vs. "Near \( x \)"

Let's consider \( f(x) = x^2 \)

At \( x = 2 \), we have \( f(2) = 4 \)

What about "near \( x = 2 \)?"

\[ \text{As } x \text{ approaches } 2 \text{ from "below", } \lim_{x \to 2^-} f(x) = 4 \]

\[ \text{As } x \text{ approaches } 2 \text{ from "above", } \lim_{x \to 2^+} f(x) = 4 \]

\[ \lim_{x \to 2} f(x) = 4 \text{ "left hand limit" } \]

\[ \lim_{x \to 2} f(x) = 4 \text{ "right hand limit" } \]

As \( x \) approaches 2 from "below", the functional values are approaching 4.
Ex: \( g(x) = \frac{x^2 - 4}{x - 2} \)  

- At \( x = 2 \), \( g(x) = \text{undefined} \)
- Near \( x = 2 \), \( g(x) \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( g(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.9</td>
<td>3.9</td>
</tr>
<tr>
<td>1.99</td>
<td>3.99</td>
</tr>
<tr>
<td>1.999</td>
<td>3.999</td>
</tr>
<tr>
<td>2.1</td>
<td>4.1</td>
</tr>
<tr>
<td>2.01</td>
<td>4.01</td>
</tr>
<tr>
<td>2.001</td>
<td>4.001</td>
</tr>
</tbody>
</table>

\( \lim_{x \to 2} g(x) = 4 \)  

\( \lim_{x \to 2^+} g(x) = 4 \) \( \text{(R.H. lim)} \)

\( \lim_{x \to 2^-} g(x) = 4 \) \( \text{(L.H. lim)} \)

At \( x = 2 \), \( g(x) \) is not defined.

As \( x \) is near \( 2 \), \( g(x) \) is near 4.

\( \lim_{x \to 2} g(x) = 4 \)

For \( \lim_{x \to a} f(x) \) to exist, we need \( \lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) \).
Ex: \( f(x) = \begin{cases} x^2, & \text{for } x < 1 \\ 2x + 3, & \text{for } x \geq 1 \end{cases} \)

What is \( f(x) \) at \( x = 1^+ \)?

What is the limit of \( f(x) \) as \( x \) approaches 1? 

\[
\begin{array}{c|c|c}
\text{L.H. lim} & \text{R.H. lim} \\
\hline
x & f(x) & x & f(x) \\
0.9 & 0.81 & 1.1 & 5.2 \\
0.99 & 0.9801 & 1.01 & 5.02 \\
0.999 & 0.998001 & 1.001 & 5.002 \\
\hline
\end{array}
\]

\[ \lim_{x \to 1^-} f(x) = 1 \]

\[ \lim_{x \to 1^+} f(x) = 5 \]

Since L.H. limit does not equal R.H. limit, the general limit does not exist ("jump")