Fourier Expansion in Orthogonal Polynomials of Several Variables

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We discuss results on Cesàro summability of the Fourier orthogonal expansion on the sphere $S^{d-1}$, on the unit ball $B^d$ and on the standard simplex $T^d$. The measures or weight functions are the classical type: $\prod_{i=1}^d |x_i|^{2\kappa_i} \, d\omega$ on $S^{d-1}$, $\prod_{i=1}^d |x_i|^{2\kappa_i}(1 - |x|^2)^{\mu-1/2}$ on $B^d$ and $\prod_{i=1}^d x_i^{\kappa_i-1/2}(1 - x_1 - \cdots - x_d)^{\mu-1/2}$ on $T^d$, where $\kappa_i \geq 0$ and $\mu \geq 0$. The main result gives the necessary and sufficient conditions for the convergence (critical index). Here is a sample of the result:

**Theorem 1** The Cesàro $(C, \delta)$ means of the Fourier orthogonal expansion with respect to the measure $\prod_{i=1}^d |x_i|^{2\kappa_i} \, d\omega$ ($\kappa_i \geq 0$) converges uniformly on $S^{d-1}$ if and only if

$$\delta > \frac{d - 2}{2} + \sum_{i=1}^d \kappa_i - \min_{1 \leq i \leq d} \kappa_i.$$ 

Moreover, the convergence holds pointwise on $S^{d-1} \setminus \{x \in S^{d-1} : x_i = 0, \text{ some } i\}$, if $\delta > (d - 2)/2$.

Part of this work is joint with Zhongkai Li.