Numerical Calculation of
the Generalized Sine and Cosine Integral
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Abstract

The generalized sine integral
\[ \text{Si}(x, \alpha) := \int_0^x \frac{\sin t}{t^\alpha} \, dt, \quad 0 < \alpha < 2, \quad x \geq 0 \]

was introduced by A. Walther in his studies on the Gibbs’s phenomenon of Fourier series.

Both functions, the generalized sine integral and the generalized cosine integral
\[ \text{Ci}(x, \alpha) := \int_0^x \frac{\cos t}{t^\alpha} \, dt, \quad 0 < \alpha < 1, \quad x \geq 0, \]

were studied with respect to their analytical behaviour in all details by E. Kreyszig.
For \( \alpha = 1 \) and \( \alpha = \frac{1}{2} \), one obtains the well-known special cases of the “ordinary” sine integral and the Fresnel integrals, respectively.

The generalized sine integral and the generalized cosine integral are used among others in the calculation of the propagation of electromagnetic waves.

The talk is on the numerical calculation by truncated “double” Chebyshev series of the generalized sine integral and the generalized cosine integral with values of \( \alpha \) in \( 0 < \alpha < 2 \) or \( 0 < \alpha < 1 \), respectively, and an arbitrary value of \( x \geq 0 \).

References:
Kreyszig, E.: Über den allgemeinen Integralsinus \( \text{Si}(x, \alpha) \), Acta mathematica 85 (1951), 118 – 181