Abstract

In 1938, H.L. Krall asked for the classification of all families of orthogonal polynomials that are also eigenfunctions of a differential operator. In this talk, I shall survey some recent work on this problem. An important issue will be to extend Krall’s problem by allowing for a doubly infinite three-term recursion relation, instead of a semi-infinite one. This extension of the problem provides a new road to the concept of the associated polynomials. It also connects the problem with the Baker-Burchnall-Chaundy theory of commutative rings of difference operators. All solutions of Krall’s problem known so far, correspond to rank 2 commutative rings of difference operators. I shall discuss a new class of rank 1 solutions to the problem, related to the rational solutions of the Toda lattice hierarchy and hyperelliptic unicursal curves. The case of higher order than three-term recursion relations will also be considered.