Expansions in series of $q$-linear sines and cosines

J. Bustoz

In this talk we describe joint work with Jose Luis Cardoso.

The functions $S_q(z)$, $C_q(z)$ defined by

$$S_q(z) = \frac{z}{1-q} \sum_{n=0}^{\infty} \frac{(-1)^n q^{n(n+1)/2} z^{2n}}{(q^2, q^3; q^2)_n},$$

$$C_q(z) = \sum_{n=0}^{\infty} \frac{(-1)^n q^{n(n-1)/2} z^{2n}}{(q^2, q; q^2)_n}$$

are solutions of the difference equations

$$\frac{\delta C_q(\omega z)}{\delta z} = -\frac{\omega}{1-q} S_q(\omega z),$$

$$\frac{\delta S_q(\omega z)}{\delta z} = \frac{\omega}{1-q} C_q(\omega z),$$

where $\delta f(z) = f(q^{1/2}z) - f(q^{-1/2}z)$. The functions $S_q(z)$ and $C_q(z)$ satisfy a discrete orthogonality. We will give bounds on the roots $S_q(z) = 0$ and examples of expansions in series of these $q$-trigonometric functions.