Write your name on each page of this exam. One page of notes and a calculator are permitted. There are six questions for a total of 60 points as indicated. While you may use calculators to evaluate integrals, you must show all of your work in setting them up to receive full credit.

1. (10 points) Compute \( \int_C xy \, dx - x \, dy \) where \( C \) is the circle of radius 2, centered at the origin, with counterclockwise orientation.

2. (10 points) Let \( \mathbf{F}(x, y, z) = xyz \mathbf{i} - x^2y \mathbf{k} \). Find the curl and the divergence of \( \mathbf{F} \). (You may use calculators to check your work, but you must state the definition of divergence and curl and carry out the computations by hand to receive full credit.)

3. Let \( \mathbf{F}(x, y) = (\sin y) \mathbf{i} + (x \cos y + \sin y) \mathbf{j} \).
   (a) (5 points) Show that \( \mathbf{F} \) is conservative by finding a potential function \( f \).
   (b) (5 points) Compute \( \int_C \mathbf{F} \cdot d\mathbf{r} \) where \( C \) is the arc of the curve \( y = \sin x \) from \((0, 0)\) to \((\pi/2, 1)\).

4. Let \( C \) be the positively oriented boundary of the region enclosed by the curves \( y = \sqrt{x}, x = 1, \) and \( y = 0 \).
   (a) (5 points) Sketch \( C \) and indicate the direction of traversal.
   (b) (5 points) Compute \( \int_C (1 + \cos x) \, dx + (x^2 + e^y) \, dy \).

5. (10 points) If \( C \) is any piecewise smooth simple closed plane curve and \( f \) and \( g \) are differentiable functions, prove that

\[
\int_C f(x) \, dx + g(y) \, dy = 0.
\]

6. (10 points) Find the surface area of the portion of the paraboloid \( z = x^2 + y^2 \) contained below the plane \( z = 4 \).