Problem 4.193 The force $F = -60i + 60j$ (lb).

(a) Determine the moment of $F$ about point $A$.
(b) What is the perpendicular distance from point $A$ to the line of action of $F$?

Solution: The position vector of $A$ and the point of action are

$r_A = 8i + 2j + 12k$ (ft), and $r_F = 4i - 4j + 2k$.

The vector from $A$ to $F$ is

$r_{AF} = r_F - r_A = (4 - 8)i + (-4 - 2)j + (2 - 12)k$

$= -4i - 6j - 10k$.

(a) The moment about $A$ is

$M_A = r_{AF} \times F = \begin{vmatrix} i & j & k \\ -4 & -6 & -10 \\ -60 & 60 & 0 \end{vmatrix} = 600k - 600j + 600i$ (ft lb)

(b) The magnitude of the moment is

$|M_A| = \sqrt{600^2 + 600^2 + 600^2} = 1039.3$ ft lb.

The magnitude of the force is $|F| = \sqrt{60^2 + 60^2} = 84.8528$ lb.

The perpendicular distance from $A$ to the line of action is

$D = \frac{1039.3}{84.8528} = 12.25$ ft.

Problem 4.196 The bar $AB$ supporting the lid of the grand piano exerts a force $F = -6i + 35j - 12k$ (lb) at $B$. The coordinates of $B$ are $(3, 4, 3)$ ft. What is the moment of the force about the hinge line of the lid (the $x$ axis)?

Solution: The position vector of point $B$ is $r_{OB} = 3i + 4j + 3k$.

The moment about the $x$-axis due to the force is

$M_x = \epsilon_3 \cdot (r_{OB} \times F) = \begin{vmatrix} 1 & 0 & 0 \\ 3 & 4 & 3 \\ -6 & 35 & -12 \end{vmatrix} = -153$ ft lb.
Problem 4.197 Determine the moment of the vertical 800-lb force about point C.

Solution: The force vector acting at A is \( \mathbf{F} = -800\mathbf{j} \) (lb) and the position vector from C to A is

\[
\mathbf{r}_{CA} = (x_A - x_C)\mathbf{i} + (y_A - y_C)\mathbf{j} + (z_A - z_C)\mathbf{k}
\]

\[
= (4 - 5)\mathbf{i} + (3 - 0)\mathbf{j} + (4 - 6)\mathbf{k} = -\mathbf{i} + 3\mathbf{j} - 2\mathbf{k} \text{ (ft)}. 
\]

The moment about C is

\[
M_C = \begin{vmatrix} i & j & k \\ -1 & 3 & -2 \\ 0 & -800 & 0 \end{vmatrix} = -1600\mathbf{i} + 0\mathbf{j} + 800\mathbf{k} \text{ (ft-lb)}. 
\]

Problem 4.198 In Problem 4.197, determine the moment of the vertical 800-lb force about the straight line through points C and D.

Solution: In Problem 4.197, we found the moment of the 800 lb force about point C to be given by

\[ M_C = -1600\mathbf{i} + 0\mathbf{j} + 800\mathbf{k} \text{ (ft-lb)}. \]

The vector from C to D is given by

\[
\mathbf{r}_{CD} = (x_D - x_C)\mathbf{i} + (y_D - y_C)\mathbf{j} + (z_D - z_C)\mathbf{k}
\]

\[
= (6 - 5)\mathbf{i} + (0 - 3)\mathbf{j} + (0 - 6)\mathbf{k} 
\]

\[ = \mathbf{i} - 3\mathbf{j} - 6\mathbf{k} \text{ (ft)}. \]

and its magnitude is

\[
|\mathbf{r}_{CD}| = \sqrt{1^2 + 6^2} = \sqrt{37} \text{ (ft)}. 
\]

The unit vector from C to D is given by

\[
\mathbf{e}_{CD} = \frac{1}{\sqrt{37}} - \frac{6}{\sqrt{37}}\mathbf{k}. 
\]

The moment of the 800 lb vertical force about line CD is given by

\[
M_{CD} = \left( \frac{1}{\sqrt{37}} - \frac{6}{\sqrt{37}}\mathbf{k} \right) \cdot (-1600\mathbf{i} + 0\mathbf{j} + 800\mathbf{k} \text{ (ft-lb)}) 
\]

\[
= \left( \frac{-1600}{\sqrt{37}} - \frac{4800}{\sqrt{37}} \right) \text{ (ft-lb)}. 
\]

Carrying out the calculations, we get \[ M_{CD} = -1052 \text{ (ft-lb)} \].