Problem 10.100 A circular plate of 1-m radius is below the surface of a stationary pool of water. Atmospheric pressure is $p_{atm} = 10^5$ Pa, and the mass density of the water is $\rho = 1000$ kg/m$^3$. Determine (a) the force exerted on the face of the plate by the pressure of the water; (b) the $x$ coordinate of the center of pressure. (See Problem 10.99.)

Solution: (a) From Problem 10.99, the pressure on the face of the plate is $F = pA$, where $p$ is the pressure at the centroid of the area.

$$F = (p_0 + \gamma z)(\pi r^2) = 375.8 \text{ kN}.$$  

(b) From Problem 10.99,

$$x_p = x + \frac{y}{p} = 2 + \frac{9810 \left( \frac{\pi r^4}{4} \right)}{p} = 2.0205 \text{ m}$$

Problem 10.116 At $A$ the main cable of the suspension bridge is horizontal and its tension is $1 \times 10^8$ lb.

(a) Determine the distributed load acting on the cable.  
(b) What is the tension at $B$?

Solution: (a) The parameter

$$a = 2 \frac{y}{x^2} = 2 \frac{300}{900} = 7.4074 \times 10^{-4}.$$  

The distributed load is

$$w = T_0a = (1 \times 10^8)(7.4 \times 10^{-4}) = 7.4074 \times 10^4 \text{ lb/ft}.$$  

(b) The tension at $B$ is

$$T_B = T_0 \sqrt{1 + a^2(900)^2} = 1.2 \times 10^8 \text{ lb}.$$
Problem 10.117 The power line has a mass of 1.4 kg/m. If the line will safely support a tension of 5 kN, determine whether it will safely support an ice accumulation of 4 kg/m.

Solution: The power line meets the conditions for a catenary. The weight density with an ice load is

\[ \gamma = (1.4 + 4)(9.81) = 52.974 \text{ N/m}. \]

The angle at the attachment point is related to the length and the parameter \( a \) by \( sa = \tan \theta \). But \( sa = \sinh(a\theta) \), and \( x \) is known. Thus the parameter can be found from

\[ a = \frac{\sinh^{-1}(\tan \theta)}{x} = \frac{\sinh^{-1}(0.2126)}{20} = 0.01055. \]

The tension at the lowest point is

\[ T_0 = \frac{w}{a} = 5021.53 = 5.02 \text{ kN}. \]

The maximum tension is

\[ T = T_0 \cosh(a\theta) = 5133.7 = 5.133 \text{ kN}. \]

Thus the line will not sustain the load.
Problem 10.121  The dam has water of depth 4 ft on one side. The width of the dam (the dimension into the page) is 8 ft. The weight density of the water is \( \gamma = 62.4 \text{ lb/ft}^3 \), and atmospheric pressure is \( p_{\text{atm}} = 2120 \text{ lb/ft}^2 \). If you neglect the weight of the dam, what are the reactions at \( A \) and \( B \)?

**Solution:**  The atmospheric pressure acts on both faces of the dam, so it is ignored. The strategy is to use the “volume” of the pressure distribution to determine the reactions. The pressure distribution is a triangle of base \( 4\gamma \) and altitude 4 ft. The force on the vertical faces of the dam is

\[
F_1 = \left( \frac{1}{2} \right) (4)(4)(\gamma)B = 3993.6 \text{ lb.}
\]

The moment about \( A \) due to the force on the vertical faces is

\[
M_1 = \left( \frac{1}{2} \right) F_1 = 5324.8 \text{ ft lb.}
\]

The force on the horizontal face of the dam is

\[
F_2 = (2)(\gamma)(2)(8) = 1996.8 \text{ lb.}
\]

The moment about \( A \) due to the force on the horizontal face is \( M_2 = 1F_2 = 1996.8 \text{ ft lb.} \) The sum of the moments about \( A \):

\[
\sum M_A = M_1 + M_2 - AB = 0, \text{ from which } B = 1830.4 \text{ lb.}
\]

The sum of the forces:

\[
\sum F_x = A_x - F_1 + B = 0, \text{ from which } A_x = 2163.3 \text{ lb to the right.}
\]

\[
\sum F_y = A_y - F_2 = 0, \text{ from which } A_y = 1996.8 \text{ lb upward.}
\]