Problem 10.105  (a) Determine the maximum bending moment in the beam and the value of $x$ where it occurs.
(b) Show that the equations for $V$ and $M$ as functions of $x$ satisfy the equation $V = dM/dx$.

**Solution:** The distributed load is a straight line function of $x$, with zero intercept and slope
\[ \frac{360}{18} = 20 \text{ lb/ft}, \]
from which $w(x) = 20x \text{ lb/ft}$. The total moment about the left end due to the loading is
\[ M = \int_0^{18} w(x) dx = \frac{20}{3} (x^2)|_0^{18} = 38880 \text{ ft lb}. \]
The reaction at the right end is
\[ \sum M = -38880 + 18B = 0, \]
from which $B = 2160 \text{ lb}$. The reaction at the left end is
\[ \sum F_y = R_y - \int_0^{15} 20x dx + B = 0, \]
from which
\[ R_y = \left(10x^2\right)_0^{18} - 2160 = 1080 \text{ lb}. \]
(a) The shear as a function of $x$ is
\[ V(x) = \sum F = R_y - \int_0^x 20x dx = 1080 - 10x^2 \text{ lb}. \]
The moment as a function of $x$ is
\[ M(x) = R_y x - \int_0^x 10x^2 dx = 1080x - \left( \frac{10}{3} \right)x^3 \text{ ft lb}. \]
The maximum moment occurs where \[ \frac{dM}{dx} = 0 = 1080 - 10x^2 = 0, \] or at $x = \sqrt{\frac{1080}{10}} = 10.39 \text{ ft}$.
The maximum moment is
\[ M_{\text{MAX}} = 1080(10.39) - \frac{10}{3} (10.39)^3 = 7482.5 \text{ ft lb}. \]
(b) The shear is
\[ \frac{dM}{dx} = \frac{d(1080x - \left( \frac{10}{3} \right)x^3)}{dx} = 1080 - 10x^2 \text{ lb}. \]
Problem 10.109 Draw the shear force and bending moment diagrams for beam ABC.

Solution: The structure as a free body: The angle of the cable at D relative to the horizontal is
\[ \theta = \tan^{-1} \left( \frac{1}{5} \right) = 14^\circ. \]

Denote the tension in the cable by \( T \). The sum of the moments about \( A \) is
\[ \sum M_A = 2T \cos \theta + 8T \sin \theta - 10(600) = 0, \]
from which \( T = 1546.2 \) lb. The sum of the forces:
\[ \sum F_y = A_y - 600 + T \sin \theta = 0, \]
from which \( A_y = 225 \) lb. The intervals as free bodies: Divide the beam into two intervals: \( 0 \leq x < 8 \), and \( 8 \leq x \leq 10 \). Interval 1: The shear force is \( V_1(x) = A_y \). The internal bending moment is \( M_1(x) = A_y x = 225x \). Interval 2: The shear force is \( V_2(x) = A_y + T \sin \theta = 600 \) lb. The sum of the moments is
\[ \sum M'(x) = M_2(x) - A_y x - (T \sin \theta)(x - 8) + 2T \cos \theta = 0, \]
from which \( M_2(x) = 600x - 6000 \) ft lb.

The diagrams are shown. The discontinuity in the moment at \( x = 8 \) is due to the moment exerted on the beam by the horizontal component of the cable tension at D. This horizontal component of tension exerts a moment that is independent of the distance \( x \) along the beam ABC. In this sense, it behaves like a couple.
Problem 10.112 Determine the internal forces and moments at B (a) if \( x = 250 \text{ mm} \); (b) if \( x = 750 \text{ mm} \).

Solution: The complete beam: The sum of the moments about \( A \) is

\[
\sum M_A = M_A - 20 + 1(40) = 0,
\]

from which \( M_A = -20 \text{ N m} \). The sum of the forces:

\[
\sum F_y = A_y + 40 = 0,
\]

from which \( A_y = -40 \text{ N} \).

\[
\sum F_x = A_x = 0
\]

The internal forces at \( x = 250 \text{ mm} \). The shear is \( V_1(x) = -40 \text{ N} \). The moment is

\[
M_1(x) = \int V_1(x) \, dx + C_1 = -40x + C_1
\]

At \( x = 0 \), \( M_1(0) = -M_A = 20 \text{ N m} \), from which \( C_1 = 20 \text{ kN} \). Thus the moment is \( M(0.25) = -40(0.25) + 20 = 10 \text{ N m} \). The internal forces at \( x = 750 \text{ mm} \). Divide the beam into two segments:

\[
0 \leq x < 0.5 \text{ (m)};
\]

and \( 0.5 \leq x < 0.75 \text{ (m)} \).

The shear in the second segment is \( V_2(x) = -40 \text{ N} \). The moment is

\[
M_2(x) = \int V_2(x) \, dx + C_2 = -40x + C_2.
\]

A known discontinuity exists in the moment at \( x = 0.5 \text{ m} \), \( M_1(0.5) = M_2(0.5) = -20 \text{ N m} \), from which \( C_2 = 40 \text{ kN} \) and the moment is

\[
M_2(x) = -40x + 40 \text{ N m} \). At \( x = 0.75 \), \( M_2(0.75) = -40(0.75) + 40 = 10 \text{ N m} \). The axial forces \( P(x) = 0 \) everywhere.
Problem 10.113 Draw the shear force and bending moment diagrams for the beam in Problem 10.112.

Solution: From the solution to Problem 10.112, the shear and bending moment are

\[ V_1(x) = -40 \text{ N}, \]

\[ M_1(x) = -40x + 20 \text{ Nm} \quad (0 \leq x < 0.5 \text{ m}), \]

and \[ V_2(x) = -40 \text{ N}, \]

\[ M_2(x) = -40x + 40 \text{ Nm} \quad (0.5 \leq x < 1 \text{ m}) \]

The shear force and bending moment diagrams are shown.