Problem 8.158  The mass of the thin homogenous plate is 4 kg. Determine its mass moment of inertia about the y axis.

Solution:  Divide the object into two parts: Part (1) is the semi-circle of radius 100 mm, and Part (2) is the rectangle 200 mm by 280 mm. The area of Part (1)

\[ A_1 = \frac{\pi R^2}{2} = 15708 \text{ mm}^2. \]

The area of Part (2) is

\[ A_2 = 280(200) = 56000 \text{ mm}^2. \]

The composite area is \( A = A_2 - A_1 = 40292 \text{ mm}^2. \) The area mass density is

\[ \rho = \frac{4}{A} = 9.9275 \times 10^{-5} \text{ kg/mm}^2. \]

For Part (1) \( x_1 = y_1 = 0. \)

\[ I_{x1} = \rho \left( \frac{1}{8} \right) \pi R^4 = 3898.5 \text{ kg-mm}^2. \]

For Part (2) \( x_2 = 100 \text{ mm}. \)

\[ I_{x2} = x_2^2 A_2 + \rho \left( \frac{1}{12} \right) (280)(200^2) = 74125.5 \text{ kg-mm}^2. \]

The composite:

\[ I_y = I_{x2} - I_{x1} = 70226 \text{ kg-mm}^2 = 0.070226 \text{ kg-m}^2. \]
Problem 8.160 The homogenous pyramid is of mass $m$. Determine its mass moment of inertia about the $z$ axis.

Solution: The mass density is

$$\rho = \frac{m}{V} = \frac{3m}{w^2h^3}.$$ 

The differential mass is $dm = \rho \omega^2 \, d\omega$. The moment of inertia of this element about the $z$ axis is

$$dI_z = \left( \frac{1}{6} \right) \omega^2 \, dm.$$ 

Noting that $\omega = \frac{w^2}{h}$, then

$$dI_z = \rho \left( \frac{w^4}{6h^4} \right) z^4 \, dz = \frac{m w^2}{2h^5} z^4 \, dz.$$ 

Integrating:

$$I_{z\text{-axis}} = \left( \frac{m w^2}{2h^5} \right) \int_0^h z^4 \, dz = \frac{1}{10} mw^2$$

Problem 8.161 Determine the mass moment of inertia of the homogenous pyramid in Problem 8.160 about the $x$ and $y$ axes.

Solution: Use the results of the solution of Problem 8.160 for the mass density. The elemental disk is $dm = \rho \omega^2 \, d\omega$. The moment of inertia about an axis through its center of mass parallel to the $x$-axis is

$$dI_x = \left( \frac{1}{12} \right) \omega^2 \, dm.$$ 

Use the parallel axis theorem:

$$I_{x\text{-axis}} = \left( \frac{1}{12} \right) \int_m \omega^2 \, dm + \int_m z^2 \, dm.$$ 

Noting that $\omega = \frac{w^2}{h}$, the integral is

$$I_{x\text{-axis}} = \frac{\rho w^4}{12h^4} \int_0^h z^4 \, dz + \frac{\omega w^2}{h^2} \int_0^h z^4 \, dz.$$ 

Integrating and collecting terms

$$I_{x\text{-axis}} = m \left( \frac{1}{20} w^2 + \frac{3}{5} h^2 \right).$$ 

By symmetry, $I_{y\text{-axis}} = I_{z\text{-axis}}$.
**Problem 8.164** Determine the mass moment of inertia of the 14-kg flywheel about the axis $L$.

![Diagram of flywheel](image)

**Solution:** The flywheel can be treated as a composite of the objects shown:

The volumes are:

- $V_1 = (150\pi r(250))^2 = 294.5 \times 10^5 \text{ mm}^3$,
- $V_2 = (150\pi r(220))^2 = 228.08 \times 10^5 \text{ mm}^3$,
- $V_3 = (50\pi r(220))^2 = 76.03 \times 10^5 \text{ mm}^3$,
- $V_4 = (50\pi r(60))^2 = 5.65 \times 10^5 \text{ mm}^3$,
- $V_5 = (100\pi r(60))^2 = 11.31 \times 10^5 \text{ mm}^3$,
- $V_6 = (100\pi r(35))^2 = 3.85 \times 10^5 \text{ mm}^3$.

The volume

$$V = V_1 - V_2 + V_3 - V_4 + V_5 - V_6$$

$$= 144.3 \times 10^5 \text{ mm}^3.$$ 

so the density is

$$\delta = \frac{14}{V} = 9.704 \times 10^{-7} \text{ kg/mm}^3.$$ 

The moment of inertia is

$$I_L = \frac{1}{4} V_1 (250)^2 - \frac{1}{4} V_2 (220)^2$$

$$+ \frac{1}{4} V_3 (220)^2 - \frac{1}{4} V_4 (60)^2$$

$$+ \frac{1}{4} V_5 (60)^2 - \frac{1}{4} V_6 (35)^2$$

$$= 536,800 \text{ kg-mm}^2$$

$$= 0.5368 \text{ kg-m}^2.$$