Problem 7.127  Determine the volume of the volume of revolution.

Solution:  The area of the triangle is
\[ A = \frac{1}{2} \times 4 \times 6 = 12 \text{ in}^2. \]
The y-coordinate of the centroid is
\[ y = 5 + \frac{1}{3} \times 4 = 6.3333 \text{ in}. \]
The volume of revolution is \[ V = 2\pi Ay = 477.52 \text{ in}^3. \]
Problem 7.133 Determine the center of mass of the homogeneous sheet of metal.

Solution: Divide the object into four parts: (1) The lower plate, (2) the left hand plate, (3) the semicircular plate, and (4) the composite plate. The areas and centroids are found by inspection:

1. \( A_1 = 9(12) = 108 \text{ in.}^2 \),
   \( x_1 = 0.5 \text{ in.}, \ y_1 = -8 \text{ in.}, \ z_1 = 6 \text{ in.} \).
2. \( A_2 = 8(12) = 96 \text{ in.}^2 \),
   \( x_2 = -4 \text{ in.}, \ y_2 = -4 \text{ in.}, \ z_2 = 6 \text{ in.} \).
3. \( A_3 = \pi(4)(12) = 150.8 \text{ in.}^2 \),
   \( x_3 = 0, \ y = \frac{2(4)}{\pi} = 2.546 \text{ in.}, \ z = 6 \text{ in.} \).

The composite area is

\[
A = \sum_{i=1}^{3} A_i = 354.796 \text{ in.}^2.
\]

The centroid for the composite:

\[
x = \frac{\sum_{i=1}^{3} A_i x_i}{A} = -0.930 \text{ in.}
\]
\[
y = \frac{\sum_{i=1}^{3} A_i y_i}{A} = -2.435 \text{ in.}
\]
\[
z = \frac{\sum_{i=1}^{3} A_i z_i}{A} = 6 \text{ in.}
\]
Problem 7.134 Determine the center of mass of the homogeneous object.

Solution: Divide the object into three parts and the composite:
(1) A triangular solid 30 mm altitude, 60 mm base, and 10 mm thick.
(2) A rectangle 60 by 70 mm by 10 mm. (3) A semicircle with radius 20 mm and 10 mm thick.

The volumes and their centroids are determined by inspection:

(1) \( V_1 = \frac{1}{2}(30)(60)(10) = 9000 \text{ mm}^3 \),
   \( x_1 = 5 \text{ mm}, \)
   \( y_1 = 10 + \frac{30}{3} = 20 \text{ mm}, \)
   \( z_1 = \frac{60}{3} = 20 \text{ mm}. \)

(2) \( V_2 = 60(70)(10) = 42000 \text{ mm}^3 \),
   \( x_2 = 35 \text{ mm}, \)
   \( y_2 = 5 \text{ mm}, \)
   \( z_2 = 30 \text{ mm}, \)

(3) \( V_3 = \frac{\pi(20)^2}{2}(10) = 6283.2 \text{ mm}^3 \),
   \( x_3 = 70 - \frac{4(20)}{3\pi} = 61.51 \text{ mm}, \)
   \( y_3 = 5 \text{ mm}, \)
   \( z_3 = 30 \text{ mm}. \)

The composite volume is \( V = V_1 + V_2 - V_3 = 44716.8 \text{ mm}^3 \). The centroid is

\[
x = \frac{V_1 x_1 + V_2 x_2 - V_3 x_3}{V} = 25.237 \text{ mm}
\]

\[
y = \frac{V_1 y_1 + V_2 y_2 - V_3 y_3}{V} = 8.019 \text{ mm}
\]

\[
z = \frac{V_1 z_1 + V_2 z_2 - V_3 z_3}{V} = 27.99 \text{ mm}
\]
Problem 7.135  
Determine the center of mass of the homogeneous object.

Solution: 
Divide the object into five parts plus the composite. 
(1) A solid cylinder with 1.5 in. radius, 3 in. long. (2) A rectangle 3 by 5 by 1 in. (3) A solid cylinder with radius 1.5 in., 2 in. long. (4) A semicircle with radius 1.5 in., 1 inch thick. (5) A semicircle with radius 1.5 in., 1 inch thick. The volumes and centroids are determined by inspection. These are tabulated:

<table>
<thead>
<tr>
<th>Part No</th>
<th>Vol, cu in.</th>
<th>x, in.</th>
<th>y, in.</th>
<th>z, in.</th>
</tr>
</thead>
<tbody>
<tr>
<td>V1</td>
<td>21.205</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>V2</td>
<td>15</td>
<td>2.5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>V3</td>
<td>14.137</td>
<td>5</td>
<td>-0.5</td>
<td>0</td>
</tr>
<tr>
<td>V4</td>
<td>3.534</td>
<td>0.6366</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>V5</td>
<td>3.534</td>
<td>4.363</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Composite</td>
<td>43.27</td>
<td>2.09</td>
<td>0.3267</td>
<td>0</td>
</tr>
</tbody>
</table>

The composite is

\[ V = \sum_{i=1}^{3} V_i - \sum_{i=4}^{5} V_i. \]

The centroid:

\[ x = \frac{\sum_{i=1}^{3} V_i x_i - \sum_{i=4}^{5} V_i x_i}{V}. \]

with a corresponding expression for y. The z-coordinate is zero because of symmetry.