Problem 5.2 The beam has a built-in support and is loaded by a 2-kN force and a 6 kN-m couple.

(a) Draw the free-body diagram of the beam.
(b) Determine the reactions at the supports.

Solution:

(a)

(b) \[ \sum F_x = 0: \quad A_x = 0 \]

\[ \sum F_y = 0: \quad A_y - 2 \text{kN} = 0 \]

\[ \sum M_A = 0: \quad M_A - (2)(2 \text{kN}) + 6 \text{kN-m} = 0 \]

\[ M_A = -2 \text{kNm} \]

\[ A_y = 0 \]

\[ A_y = 2 \text{kN} \]
Problem 5.140  Determine the reactions at A and B.

Solution: From the free body diagram at the right, the equations of equilibrium are

\[ \sum F_x = 400 \cos(30^\circ) + A_x = 0. \]

\[ \sum F_y = A_y + B_y - 400 \sin(30^\circ) = 0. \]

and \[ \sum M_A = (0.5B_y - (0.22)(400) \cos 30^\circ) = 0. \]

Solving these three equations in the three unknowns, we get

\[ A_x = -346.4 \text{ N}, \]

\[ A_y = 47.6 \text{ N}, \]

and \[ B_y = 152.4 \text{ N}. \]
Problem 5.141 Palentologists speculate that the stegosaurus could stand on its hind limbs for short periods to feed. Based on the free-body diagram shown and assuming that \( m = 2000 \) kg, determine the magnitudes of the forces \( B \) and \( C \) exerted by the ligament — muscle brace and vertical column, and determine the angle \( \alpha \).

Solution: Take the origin to be at the point of application of the force \( C \). The position vectors of the points of application of the forces \( B \) and \( W \) are:

\[
\mathbf{r}_B = -415 \mathbf{i} + 160 \mathbf{j} \text{ (mm)},
\]
\[
\mathbf{r}_W = 790 \mathbf{i} + 580 \mathbf{j} \text{ (mm)}.
\]

The forces are

\[
\mathbf{C} = C (\mathbf{i} \cos(90^\circ - \alpha) + \mathbf{j} \sin(90^\circ - \alpha))
\]
\[
= C (\mathbf{i} \sin \alpha + \mathbf{j} \cos \alpha).
\]

\[
\mathbf{B} = B (\mathbf{i} \cos(270^\circ - 22^\circ) + \mathbf{j} \sin(270^\circ - 22^\circ))
\]
\[
= B (-0.3746 \mathbf{i} - 0.9272 \mathbf{j}).
\]

\[
\mathbf{W} = -2(9.81) \mathbf{j} = -19.62 \mathbf{j} \text{ (kN)}.
\]

The moments about \( C \),

\[
\mathbf{M}_C = \begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
-415 & 160 & 0 \\
-0.3746B & -0.9272B & 0 \\
790 & 580 & 0 \\
0 & -19.62 & 0
\end{vmatrix}
\]

\[
= 444.72B - 15499.8 = 0,
\]

from which

\[
B = \frac{15499.8}{444.72} = 34.85 \text{ kN}.
\]

The sums of the forces:

\[
\sum \mathbf{F}_x = (C \sin \alpha - 0.3746B) \mathbf{i} = 0,
\]

from which \( C \sin \alpha = 13.06 \text{ kN} \).

\[
\sum \mathbf{F}_y = (C \cos \alpha - 0.9272B - 19.62) \mathbf{j} = 0,
\]

from which \( C \cos \alpha = 51.93 \text{ kN} \).

The angle \( \alpha \) is

\[
\alpha = \tan^{-1} \left( \frac{13.06}{51.93} \right) = 14.1^\circ.
\]

The magnitude of \( C \),

\[
C = \sqrt{13.06^2 + 51.93^2} = 53.55 \text{ kN}.
\]
Problem 5.144  Assume that the force exerted on the head of the nail by the hammer is vertical, and neglect the hammer's weight.

(a) Draw the free-body diagram of the hammer.

(b) If $F = 10$ lb, what are the magnitudes of the forces exerted on the nail by the hammer and the normal and friction forces exerted on the floor by the hammer?

Solution: Denote the point of contact with the floor by $B$. The perpendicular distance from $B$ to the line of action of the force is 11 in. The sum of the moments about $B$ is $M_B = 11F - 2F_N = 0$, from which the force exerted by the nail head is $F_N = \frac{11F}{2} = 5.5F$. The sum of the forces:

$$\sum F_X = -F \cos 25 + H_x = 0,$$

from which the friction force exerted on the hammer is $H_x = 0.9063F$.

$$\sum F_Y = N_H - F_N + F \sin 25 = 0.$$

from which the normal force exerted by the floor on the hammer is $N_H = 5.077F$.

If the force on the handle is

$F = 10$ lb,

then $F_N = 55$ lb,

$H_x = 9.063$ lb,

and $N_H = 50.77$ lb.