Exam: It is **not** permitted to discuss the content of this exam with others. All your work must be shown; be neat and mark your answers. (10/3 points each problem).

1. The collar $A$ slides on a smooth vertical bar. The masses are $m_A = 20$ kg and $m_B = 10$ kg, and the spring constant is $k = 360$ N/m. When $h = 0.2$ m, the spring is unstretched. Determine the value of $h$ when the system is in equilibrium.
spring constant \( k = 360 \text{ N/m} \). When \( h = 0.2 \text{ m} \), the spring is unstretched. Determine the value of \( h \) when the system is in equilibrium.

**Solution:** The diagram of the triangle shows that the amount of stretch in the spring is given by \( \delta = c - c_0 \).

From the free body diagrams of masses \( A \) and \( B \), the equations of equilibrium are:

For \( A \):
\[
\sum F_x = T \cos \alpha - N = 0
\]
and
\[
\sum F_y = T \sin \alpha - m_A g = 0
\]
and for \( B \):
\[
\sum F_x = T - k(c - c_0) - m_B g = 0
\]

From the geometry, we know that
\[
\frac{c_0}{c} = \frac{h_0}{0.25} = \frac{h}{h_0}
\]
and that
\[
\sin \alpha = \frac{h}{c}
\]

Substituting in the known values, we get a set of equations which must be solved either by iteration or by graphical methods. Using an iterative solution, we get \( h = 0.218 \text{ m} \).

A graphical solution strategy can be easily employed. Once we know a value for \( h \), we can calculate the values of all of the forces in the Problem. The only equation which will not be satisfied is the \( y \)-direction equilibrium equation for mass \( A \). We see from the free body diagram for \( A \) that the weight of \( A \) must be balanced by the vertical component of \( T \) for equilibrium. We need only calculate the vertical component of the force \( T \) acting on \( A \) and compare this to the weight of \( A \). The results of such a comparison are shown here. Note that the sloping line (the vertical component of \( T \)) crosses the horizontal line (the weight of \( A \)) at \( h \approx 0.217 \text{ m} \). This is very close to our previous result.
2. The tension in the cable \( AB \) is 100 lb, and the tension in the cable \( CD \) is 60 lb. Suppose that you want to replace these two cables by a single cable \( EF \) so that the force exerted on the wall at \( E \) is equivalent to the two forces exerted by cables \( AB \) and \( CD \) on the walls at \( A \) and \( C \). What is the tension in cable \( EF \), and what are the coordinates of points \( E \) and \( F \)?
Problem 4.165  The tension in cable $AB$ is 100 lb, and the tension in cable $CD$ is 60 lb. Suppose that you want to replace these two cables by a single cable $EF$ so that the force exerted on the wall at $E$ is equivalent to the two forces exerted by cables $AB$ and $CD$ on the walls at $A$ and $C$. What is the tension in cable $EF$, and what are the coordinates of points $E$ and $F$?

**Solution:** The position vectors of the points $A$, $B$, $C$, and $D$ are

$$r_A = 0i + 6j + 6k,$$
$$r_B = 3i + 0j + 8k,$$
$$r_C = 4i + 6j + 0k,$$ and
$$r_D = 7i + 0j + 2k.$$

The unit vectors parallel to the cables are obtained as follows:

$$r_{AB} = r_B - r_A = 3i - 6j + 2k,$$
$$|r_{AB}| = \sqrt{3^2 + (-6)^2 + 2^2} = 7,$$
from which

$$e_{AB} = \frac{r_{AB}}{|r_{AB}|} = \frac{3i - 6j + 2k}{7}.$$

$$r_{CD} = r_D - r_C = 3i - 6j + 2k,$$
$$|r_{CD}| = \sqrt{3^2 + (-6)^2 + 2^2} = 7,$$
from which

$$e_{CD} = \frac{r_{CD}}{|r_{CD}|} = \frac{3i - 6j + 2k}{7}.$$

Since $e_{AB} = e_{CD}$, the cables are parallel. To duplicate the force, the single cable $EF$ must have the same unit vector.

The force on the wall at point $A$ is

$$F_A = 100e_{AB} = 42.86i - 85.71j + 28.57k \text{ (lb)}.$$

The force on the wall at point $C$ is

$$F_C = 60e_{CD} = 25.72i - 51.43j + 17.14k \text{ (lb)}.$$

The total force is

$$F_{EF} = 68.58i - 137.14j + 45.71k \text{ (lb)},$$

$$|F_{EF}| = 160 \text{ lb}.$$

For the systems to be equivalent, the moments about the origin must be the same. The moments about the origin are

$$\sum M_0 = (r_A \times F_A) + (r_C \times F_C) =
\begin{bmatrix}
\text{i} & \text{j} & \text{k}
\end{bmatrix}
\begin{bmatrix}
42.86 & -85.71 & 28.57 \\
6 & 6 & 0 \\
25.72 & -51.43 & 17.14
\end{bmatrix}
= 788.57i + 188.57j - 617.14k.$$

This result is used to establish the coordinates of the point $E$. For the one cable system, the end $E$ is located at $x = 0$. The moment is

$$M_1 = r \times F_{EF} =
\begin{bmatrix}
\text{i} & \text{j} & \text{k}
\end{bmatrix}
\begin{bmatrix}
68.58 & -137.14 & 45.71 \\
0 & y & z \\
6.88 & 68.58 & 2.75
\end{bmatrix}
= (45.71y + 137.14z)i + (68.58z - y)j + (68.58y)k
= 788.57i + 188.57j - 617.14k,$$

from above. From which

$$y = \frac{617.14}{68.58} = 8.999 \ldots \text{ ft},$$

$$z = \frac{188.57}{68.58} = 2.75 \text{ ft}.$$

Thus the coordinates of point $E$ are $(0, 9, 2.75)$ ft. The coordinates of the point $F$ are found as follows: Let $L$ be the length of cable $EF$. Thus, from the definition of the unit vector, $y_F = z_F = Le_y$, with the condition that $y_F = 0$, $L = \frac{9}{5}$ = 10.5 ft. The other coordinates are $x_F = x_E = Le_x$, from which $x_F = 0 + 10.5(0.4286) = 4.5$ ft

$$z_F = z_E = Le_z,$$ from which $z_F = 2.75 + 10.5(0.2857) = 5.75$ ft. The coordinates of $F$ are $(4.5, 0, 5.75)$ ft.
3. System 1 consist of two forces and a couple. Suppose that you want to represent it by a wrench (system 2). Determine the force $\vec{F}$, the couple $\vec{M}_p$, and the coordinates $x$ and $z$ where the line of action of $\vec{F}$ intersects the $x$–$z$ plane.
Problem 4.173 System 1 consists of two forces and a couple. Suppose that you want to represent it by a wrench (system 2). Determine the force $\mathbf{F}$, the couple $M_p$, and the coordinates $x$ and $z$ where the line of action of $\mathbf{F}$ intersects the $x-z$ plane.

Solution: The sum of the forces in System 1 is $\mathbf{F} = 300\mathbf{j} + 600\mathbf{k}$ (N). The equivalent force in System 2 must have this value. The unit vector parallel to the force is $\mathbf{e}_F = 0.4472\mathbf{j} + 0.8994\mathbf{k}$. The sum of the moments in System 1 is

$$M = 600(3\mathbf{i} + 300\mathbf{j})k + 1000\mathbf{i} + 600\mathbf{j}$$

$$= 2800\mathbf{i} + 600\mathbf{j} + 1200\mathbf{k} \text{ (kN m)}.$$

The component parallel to the force is

$$M_p = 599.963\mathbf{j} + 1199.93\mathbf{k} \text{ (kN-m)} = 600\mathbf{j} + 1200\mathbf{k} \text{ (kN-m)}.$$

The normal component is $M_N = M - M_p = 2800\mathbf{i}$. The moment of the force

$$M_N = \begin{bmatrix} i & j & k \\ x & 0 & z \\ 0 & 300 & 600 \end{bmatrix} = -300z\mathbf{i} - 600x\mathbf{j} + 300y\mathbf{k} = 2800\mathbf{i},$$

from which

$$x = 0, z = \frac{2800}{300} = -9.333 \text{ m}.$$