CONFORMALLY RECURRENT
SEMI-RIEMANNIAN MANIFOLDS

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ABSTRACT. In this paper we give a complete classification of conformally recurrent semi-Riemannian manifolds with harmonic conformal curvature tensor and to give another generalization of conformally symmetric Riemannian manifolds. Moreover, we give a nontrivial example which is neither locally symmetric nor conformally flat.

1. Introduction. Let us denote by $M$ an $n(\geq 4)$-dimensional semi-Riemannian manifold with semi-Riemannian metric $g$ and Riemannian connection $\nabla$ and let $R$, respectively $S$ or $r$, be the Riemannian curvature tensor, respectively the Ricci tensor or the scalar curvature, on $M$.

It is said to be conformally recurrent if the conformal curvature tensor $C$ with components $C_{ijkl}$ so that

\[
C_{ijkl} = R_{ijkl} - \frac{1}{n-2} \left( S_{il} g_{jk} - S_{ik} g_{jl} + S_{jkl} g_{il} - S_{jil} g_{jk} \right) + \frac{r}{(n-1)(n-2)} \left( g_{il} g_{jk} - g_{ik} g_{jl} \right)
\]

is recurrent, i.e., there is a 1-form $\alpha$ such that $\nabla C = \alpha \otimes C$, where $R_{ijkl}, S_{ij}$ and $g_{ij}$ are components of $R$, $S$ and $g$ on $M$. In particular, it is said to be conformally symmetric if $\nabla C = 0$. As is easily seen, the class of conformally recurrent semi-Riemannian manifolds includes all the classes of conformally symmetric, conformally flat and locally symmetric semi-Riemannian manifolds. Among them such kind of Riemannian manifolds are studied by Besse [2], Ryan [12], Simon [13], Weyl [15, 16], Yano [17], Yano and Bochner [18], for example.

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