BIFURCATIONS OF BOUNDED SOLUTIONS OF ORDINARY DIFFERENTIAL EQUATIONS DEPENDING ON A PARAMETER

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ABSTRACT. In this paper, using the notion of an isolated invariant set and an isolating block, an existence criterion of bifurcation points of nonstationary bounded solutions for planar systems depending on a parameter is given.

1. Introduction. Consider the one-parameter family of differential systems in $\mathbb{R}^n$

\begin{equation}
\frac{dx}{dt} = F(x, \lambda).
\end{equation}

Let $F : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^n$ be continuous and assume that, for each $\lambda \in \mathbb{R}$, the solution of an initial value problem is unique.

Each zero of $F$ is called a stationary solution of (1.1). Clearly, if $(x_0, \lambda_0)$ satisfies $F(x_0, \lambda_0) = 0$, then $x_0$ is a critical point of the $\lambda = \lambda_0$ system (1.1). In this paper we shall investigate bifurcation points of nonstationary bounded solutions of (1.1), where a bounded solution means that it is bounded both in the forward and backward time directions.

Definition 1.1 [3]. A point $(x_0, \lambda_0) \in \mathbb{R}^n \times \mathbb{R}$ is said to be a bifurcation point of nonstationary bounded solutions of the system (1.1) if for any open neighborhood $U$ of $(x_0, \lambda_0)$ there is a nonstationary solution of (1.1) included in $U$.

It follows directly from Definition 1.1 that if $(x_0, \lambda_0)$ is a bifurcation point, then $x_0$ has to be a critical point of the $\lambda = \lambda_0$ system (1.1).