Exercise 1. Let $T$ be a closed operator on a Hilbert space $\mathcal{H}$. Prove that the spectrum of $T$ is closed.

Exercise 2. A character of $\mathbb{R}$ is a continuous function $\phi: \mathbb{R} \to \mathbb{T}$ such that
\[ \phi(s + t) = \phi(s) \phi(t) \]
for all $s, t \in \mathbb{R}$.

Use Stone’s Theorem to prove that for every character $\phi$ of $\mathbb{R}$ there exists $x \in \mathbb{R}$ such that
\[ \phi(t) = e^{itx} \]
for all $t \in \mathbb{R}$. 