Throughout this discussion, we have \( T \in B(H) \), where \( H \) is a Hilbert space over \( \mathbb{C} \).

A closed subspace \( M \) of \( H \) is \textit{invariant} under \( T \) if \( T(M) \subseteq M \), and \textit{reduces} \( T \) if both \( M \) and \( M^\perp \) are \( T \)-invariant.

**Proposition.** Let \( P \) be the projection onto a closed subspace \( M \) of \( H \). Then:

1. \( M \) is \( T \)-invariant if and only if \( TP = PTP \);
2. \( M \) reduces \( T \) if and only if \( TP = PT \), if and only if \( M \) is invariant under both \( T \) and \( T^* \).

**Proposition.** Let \( \{M_i\}_{i \in I} \) be a pairwise orthogonal set of closed subspaces of \( H \). Then \( \overline{\text{span}}_{i} M_i \)

1. is \( T \)-invariant if each \( M_i \) is, and
2. reduces \( T \) if each \( M_i \) does.

**Proposition.** If \( T \) is normal and \( \lambda \) is an eigenvalue of \( T \), then \( \ker(T - \lambda) \) reduces \( T \).