Let $X$ be a topological vector space and $Y$ a closed subspace. Let $Q: X \to Y$ denote the quotient map $Qx = x + Y$. The quotient space $X/Y$ is given the quotient topology generated by $Q$, that is, the strongest topology making $Q$ continuous.

**Proposition.** Let $X$ be a topological vector space and $Y$ a closed subspace. Then:

1. The quotient space $X/Y$ is a topological vector space, and $Q$ is open.
2. If $B$ is a local base for $X$, then $\{Q(U) : U \in B\}$ is a local base for $X/Y$.
3. If $X$ is locally convex, locally bounded, metrizable, normable, an $F$-space, Fréchet, or Banach, then so is $X/Y$.

**Corollary.** Let $Y$ and $Z$ be subspaces of a topological vector space. If $Y$ is closed and $Z$ is finite dimensional, then $Y + Z$ is closed.

**Proposition.** Let $X$ and $Y$ be topological vector spaces, let $N$ be a closed subspace of $X$, and let $T: X \to Y$ be linear. Suppose $N \subset \ker T$. Then there exists a unique linear map $S$ making the diagram

$$
\begin{array}{ccc}
X & \xrightarrow{T} & Y \\
\downarrow{Q} & & \downarrow{S} \\
X/N & \nearrow{s} \\
\end{array}
$$

commute (where $Q$ is the quotient map), and moreover $T$ is continuous or open if and only if $S$ is.