Proposition 1. If $X_1, \ldots, X_n$ are second countable, then

$$\bigotimes_1^n B_{X_i} = B_X,$$

where $X = \prod_1^n X_i$.

Proof. LHS is generated by $\pi_i^{-1}(U)$ for $i = 1, \ldots, n$ and $U$ open in $X_i$. These sets are open in $X$, hence contained in RHS. Thus LHS $\subseteq$ RHS.

To see that LHS $\supseteq$ RHS, for each $i$ choose a countable base $\mathcal{E}_i$ for $X_i$. Then $\{\bigotimes_1^n E_i : E_i \in \mathcal{E}_i\}$ is a countable base for $X$, and is contained in LHS. Every open set in $X$ is a countable union from this base, hence is a member of LHS. This suffices because RHS is generated by the open sets in $X$. \qed