Integrable functions — Extra Exercises

1. Give an example where the inequality in Fatou’s Lemma is strict.

2. Let \((X, \mathcal{M}, \mu)\) be a measure space, and let \(f, g : X \to \mathbb{R}\) be measurable. Prove that if \(g \in L^1\) and \(|f| \leq |g|\), then \(f \in L^1\).

3. Prove Proposition 6, Part (1) in Lecture 11 on Integrable Functions.