Implicit functions — Exercises

1. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be $C^1$ and $(a, b) \in \mathbb{R}^2$. Assume $f'(a, b) \neq 0$. Use the Implicit Function Theorem to prove that $f$ is not 1-1.

2. Let $E \subset X \times Y$, $f : E \to Y$, and $(a, b) \in E$. Assume that $f$ is $C^1$ and $D_2 f(a, b)$ is invertible. Prove that $f(a, b)$ is in the interior of the range of $f$.
   Hint: use the proof of the Implicit Function Theorem.

3. Let $E \subset X$ and $f : E \to Y$. Assume that $f$ is $C^1$ and $f'(x)$ is onto for all $x \in E$. Prove that $f(E)$ is open.

4. Let $U \subset X$ be open and $f : U \to Y$ be $C^1$. Prove that the set
   $\{x \in U : f'(x) \text{ is onto}\}$
   is open.