1. Let \( f: \mathbb{R} \to \mathbb{R} \) be monotone. Prove that if the range of \( f \) is dense in \( \mathbb{R} \), then \( f \) is onto.

2. The Power Rule (Corollary 13.8 in the Lecture Notes) tells us how to differentiate \( x^n \) for all \( n \in \mathbb{Z} \). Indicate (without giving complete proofs—just summarize the arguments) how the Inverse Function Theorem (Theorem 13.19) can be used to extend the Power Rule to rational exponents. Be careful to distinguish between rationals with odd denominators and even denominators—in the first case the formula should hold for all \( x \neq 0 \), and in the latter case only for \( x > 0 \). You should first deal with the case \( x^{1/n} \) (for \( n \in \mathbb{Z} \)), and for this you should first of all show that \( x \mapsto x^n \) is invertible, using the Mean Value Theorem and its consequences. You should do this first on the interval \((0, \infty)\), and when \( n \) is odd you can then do it on the interval \((-\infty, 0)\). Again, try to be brief, just summarizing the arguments.

3. Prove Darboux’s Intermediate Value Theorem for Derivatives: if \( f: (a, b) \to \mathbb{R} \) is differentiable and \( a < s < t < b \), then for every \( m \) strictly between \( f'(s) \) and \( f'(t) \) there exists \( c \in (s, t) \) such that \( f'(c) = m \). Hint: first show that if \( f'(s) < 0 \) and \( f'(t) > 0 \) then there exists \( c \in (s, t) \) such that \( f'(c) = 0 \), then for the general case consider \( g(x) = f(x) - mx \).