MAT 472 HOMEWORK 6

For the first two problems, use the following definitions. For any set $S$, let $B(S)$ denote the set of bounded real-valued functions on $S$. Give $B(S)$ the usual (pointwise) vector space operations. For $f \in B(S)$ define

$$
\|f\| := \sup\{|f(x)| : x \in S\}.
$$

1. Prove that the above gives a norm on $B(S)$, and that $B(S)$ is then a Banach space.

2. Prove that the metric space $B(\mathbb{R})$ is not separable. Hint: one way to do this involves showing $B(\mathbb{R})$ contains an uncountable subset with no cluster point.

3. (This problem is not related to the first two.) Let $A$ be a connected subset of a metric space $X$, and let $A \subseteq B \subseteq \bar{A}$. Prove that $B$ is connected. Hint: you might find it helpful to do it first under the assumption that $B = X$.

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