MAT 472 HOMEWORK 2

1. Let

\[ E = (\mathbb{Q} \cap [0, 1]) \cup \{2 + \frac{1}{n} : n \in \mathbb{N}\} \cup [4, 5) \cup (5, 6). \]

Find \( \bar{E} \) and \( E' \).

2. Let \( A \) be a subset of a metric space \( X \). The interior of \( A \), denoted \( A^\circ \), is defined as the union of the family of all open subsets of \( X \) which are contained in \( A \). Prove that \( x \in A^\circ \) if and only if there exists \( \epsilon > 0 \) such that \( B_\epsilon(x) \subseteq A \).

3. Let \( A \) be a subset of a metric space \( X \). The boundary of \( A \), denoted \( \partial A \), is defined as \( \bar{A} \setminus A^\circ \). Prove that \( x \in \partial A \) if and only if for all \( \epsilon > 0 \) we have both \( B_\epsilon(x) \cap A \neq \emptyset \) and \( B_\epsilon(x) \setminus A \neq \emptyset \).