MAT 472 Midterm

October 17, 1997

Show all your work. Please write on only one side of each sheet of paper. Write your name on every sheet, including the exam sheet. All problems have equal credit. Be as thorough as you can in your solutions.

1. Let

\[ E = (\mathbb{Q} \cap [0, 1]) \cup \left\{ 2 + \frac{1}{n} : n \in \mathbb{N} \right\} \cup [4, 5) \cup (5, 6). \]

Find \( E^c, E', E^o, (E)^o, \) and \( E^c. \)

2. Let \( A \) and \( B \) be nonempty, bounded sets of real numbers, and let \( A + B = \{ x + y : x \in A, y \in B \} \). Prove that

\[ \sup(A + B) = \sup A + \sup B. \]

3. Let \( E \) be the range of a sequence \( \{x_n\} \) in a metric space \( X. \)

(a) Prove that every limit point of \( E \) is a limit of a subsequence of \( \{x_n\}. \)

(b) Show by example that a limit of a subsequence of \( \{x_n\} \) need not be a limit point of \( E \). Be sure to justify your assertions.

4. Let \( \{a_n\} \) be a sequence of real numbers. Prove that if the series \( \sum a_n^2 \) converges, then the series \( \sum \frac{a_n}{n} \) converges absolutely.