1. Find a real orthogonal matrix whose first row is $\begin{pmatrix} 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \end{pmatrix}$.

2. Is there a linear change of coordinates (i.e., a change of basis) in $\mathbb{R}^2$ which transforms the quadratic form $x_1^2 + 4x_1 x_2 + x_2^2$ into $y_1^2 + y_2^2$? Explain.

3. Let $A$ be a invertible matrix with integer entries. Prove that $A^{-1}$ has integer entries if and only if $\det(A) = \pm 1$.

4. Let $T$ be a self-adjoint linear operator on a finite-dimensional complex inner product space $V$. Use the Spectral Theorem to prove that there exists a self-adjoint linear operator $S$ on $V$ such that $S^3 = T$.

5. Construct as many pair-wise nonsimilar complex matrices as possible, each of which has characteristic polynomial $(2-t)^4(3-t)^3$ and minimal polynomial $(t-2)^2(t-3)^2$. State any relevant theorem(s) and explain your reasoning.

6. Let $A$ be a complex matrix such that $A^2 = A$. Prove that the trace of $A$ equals the rank of $A$.

7. Define $T: P_3 \to P_3$ by

$$T(c_0 + c_1 x + c_2 x^2 + c_3 x^3) = c_0 + c_1 x + c_1 x + (-c_1 + c_2 + c_3) x^2 + c_3 x^3.$$  

Find a basis $\beta$ such that the matrix of $T$ with respect to $\beta$ is in Jordan form.