4. Upper and Lower Bounds

Exercise 4.1. Let

\[ A := \left\{ \frac{x}{x+1} \bigg| x \in (0, 1) \right\}. \]

Use only the elementary properties of inequalities to find, with full justification:

(a) \( \sup A \);

(b) \( \inf A \).

Exercise 4.2. Prove that if \( A, B \subset \mathbb{R} \) are nonempty and bounded above, then

\[ \sup(A \cup B) = \max\{\sup A, \sup B\}. \]

Exercise 4.3. Prove that if \( A, B \subset \mathbb{R} \) are nonempty and bounded above, then

\[ \sup(A + B) = \sup A + \sup B, \]

where \( A + B = \{x + y \mid x \in A, y \in B\} \).