Definition. Let \( A \subset \mathbb{R}, \) let \( f: A \to \mathbb{R}, \) let \( u \in \mathbb{R}, \) and suppose \( A \) is unbounded above. Then \( u \) is the limit of \( f \) at \( \infty, \) written \( u = \lim_{x \to \infty} f(x), \) if for all \( \epsilon > 0 \) there exists \( a \in \mathbb{R} \) such that for all \( x \in A, \)
\[
    \text{if } x > a \text{ then } |f(x) - u| < \epsilon.
\]
Similarly for limit at \(-\infty\).

All properties of limits continue to hold for limits at \( \pm \infty. \)

Definition. Let \( A \subset \mathbb{R}, \) let \( f: A \to \mathbb{R}, \) and let \( t \) be a cluster point of \( A. \) Then \( f \) goes to \( \infty \) at \( t, \) written \( \lim_{x \to t} f(x) = \infty, \) if for all \( a \in \mathbb{R} \) there exists \( \delta > 0 \) such that for all \( x \in A \setminus \{t\}, \)
\[
    \text{if } |x - t| < \delta \text{ then } f(x) > a.
\]
Similarly for goes to \(-\infty. \) Also similarly for one-sided infinite limits.

Definition. Let \( A \subset \mathbb{R}, \) let \( f: A \to \mathbb{R}, \) and suppose \( A \) is unbounded above. Then \( f \) goes to \( \infty \) at \( \infty, \) written \( \lim_{x \to \infty} f(x) = \infty, \) if for all \( a \in \mathbb{R} \) there exists \( b \in \mathbb{R} \) such that for all \( x \in A, \)
\[
    \text{if } x > b \text{ then } f(x) > a.
\]
Similarly for goes to \(-\infty, \) at either \( \infty \) or \(-\infty. \)

Most of the properties of limits continue to hold, with appropriate modifications, for infinite limits.