Name:

- Time limit: 90 minutes (1.5 hours)
- Be sure to show all your work!
- No books, notes or calculators of any kind are permitted on the test.
- Write each solution in the space provided, not on scratch paper.
- If you need more room, write on the back of the page. If you still need more room, ask for scratch paper.
- Be sure to give reasons for your answers (except where explicitly told otherwise)!
- Points will probably be taken off if you do not give a clear indication of the method and write your solution neatly, in order, and clearly indicate your final answer.
- Points may be taken off for a correct answer with a mistake in the supporting work.
1. State the following theorems:
   (a) Fundamental Theorem of Calculus (either form)
   (b) Ratio Test for series
2. In each part of this problem, give an example with the indicated properties. You do not need to give any reasons.

(a) a function $f : [0, 1] \rightarrow \mathbb{R}$ which is integrable but not continuous

(b) a sequence $(f_n)$ of functions on $[0, 1)$ such that $f_n \rightarrow 0$ pointwise but not uniformly

(c) a power series $\sum_{n=0}^{\infty} c_n x^n$ which converges at $-1$ but diverges at $1$
3. Let $a < c < b$, and let $f : [a, b] \to \mathbb{R}$ be integrable. Suppose that $\int_{a}^{c} f > 0$ and $\int_{a}^{b} f < 0$. Prove that there exists $t \in (c, b)$ such that $\int_{a}^{t} f = 0$. 
Let \( \sum_{n=0}^{\infty} c_n(x - a)^n \) be a power series, and let \( 0 < s \leq t < \infty \). Suppose \( s \leq c_n \leq t \) for all \( n \in \mathbb{N} \). Prove that the power series \( \sum_{n=0}^{\infty} c_n(x - a)^n \) has radius of convergence 1.
5. Let \( A \subseteq \mathbb{R} \), let \( f : A \to \mathbb{R} \), and let \((f_n)\) be a sequence of functions from \( A \) to \( \mathbb{R} \). Assume that \( f_n \to f \) uniformly, and that for each \( n \in \mathbb{N} \) the function \( f_n \) is bounded. Prove that \( f \) is bounded.