• Be sure to show all your work!
• Be sure to give reasons for your answers (except where explicitly
told otherwise)!
• Points will probably be taken off if you do not give a clear indi-
cation of the method and write your solution neatly, in order,
and clearly indicate your final answer.
• Points may be taken off for a correct answer with a mistake in
the supporting work.
• No books, notes or calculators of any kind are permitted on the
test.
• Write each solution in the space provided, not on scratch paper.
• If you need more room, write on the back of the page.
1. (a) Complete the following definition: the union of a family \( \{A_i\}_{i \in I} \) of sets is . . .

(b) State the Completeness Axiom of the real numbers.
2. Prove that for all sets $A$ and $B$, if $A = A \cap B$ then $A \subseteq B$. 
3. In each part of this problem, find the indicated quantity. You do not need to give reasons. Note: in parts (a)–(b) the parentheses denote open intervals, not ordered pairs.

(a) \( \bigcap_{x \in (0, \infty)} (-x, x) \)

(b) \( \bigcup_{n=1}^{\infty} (-n, n) \)

(c) \( \inf \left\{ \frac{1}{n} : n \in \mathbb{N} \right\} \)
4. Let $f: A \to B$ and $C, D \subseteq A$. Prove that if $A = C \cup D$, $f|C$ and $f|D$ are both 1-1, and $f(C) \cap f(D) = \emptyset$, then $f$ is 1-1.
5. In each part of this problem, either give an example of the indicated phenomenon or state that it is impossible. You do not need to give reasons.

   (a) a function $f : A \to B$ which is onto, where $A$ is uncountable and $B$ is countable.

   (b) a function $f : A \to B$ which is 1-1, where $A$ is uncountable and $B$ is countable.

   (c) a function $f : \mathbb{N} \to \mathbb{N}$ which is 1-1 and not onto.