MAT 371 Homework 8
Instructor: John Quigg

Due: Friday, March 22

1. Define $f : [0,1] \rightarrow \mathbb{R}$ by

$$f(x) = \begin{cases} 
1 & \text{if } x = 0 \\
0 & \text{if } 0 < x \leq 1.
\end{cases}$$

(a) Find the set of all upper sums of $f$ (i.e., the set of numbers of the form $U(P)$ for $P$ a partition of $[0,1]$).

(b) Find the set of all lower sums of $f$.

2. Let $f : [0,1] \rightarrow \mathbb{R}$ be bounded, and suppose $f$ is continuous except at 0. Prove that $f$ is integrable.

3. Let $f$ be integrable on $[a,b]$. Let $(P_n)$ be a sequence of partitions of $[a,b]$ such that $\|P_n\| \rightarrow 0$, and for each $n \in \mathbb{N}$ let $S_n$ be a Riemann sum for $f$ associated to the partition $P_n$. Prove that $S_n \rightarrow \int_a^b f$. 