MAT 371 Test 1  Thursday, February 10
Instructor: John Quigg

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Honor Statement: I have not received, nor will I give, any information regarding this exam before it is returned graded by the instructor.

Signed ________________________________

- Time limit: 1.5 hours (90 minutes).
- No notes, books, or calculators.
- You must show all your work to receive credit.
- Your solutions must be complete and organized, and your final answer must be clearly indicated.
- Write your solutions on blank pages obtained from the Testing Clerk, on one side only, leaving a 1-inch margin on all sides, and put your name at the top right corner of every page.
- This exam has 2 pages.
- All problems have equal credit.
- Turn in your exam face up, with the exam question page(s) on top. If you do not have a stapler, the exam clerk will staple the exam; do not fold the corner of the exam over.
1. Complete the following definitions:
   (a) If \((x_n)\) is a sequence of real numbers and \(x \in \mathbb{R}\), then \((x_n)\) converges to \(x\) if ...
   (b) If \(A \subseteq \mathbb{R}\) and \(x \in \mathbb{R}\), then \(x\) is the \textit{infimum} of \(A\) if ...

2. Find all cluster points of the set \(\mathbb{Q} \cap (0, 1)\), and prove that your answer is correct.

3. Let \(f: A \rightarrow B\) and let \(C \subseteq A\) and \(D \subseteq A\). Prove that \(f(C \cup D) \subseteq f(C) \cup f(D)\).

4. In each part of this problem, answer True or False (and write the word, not merely a letter!). Also, give a \textit{brief} reason.
   (a) There exists a sequence whose range is \(\mathbb{Q}\).
   (b) There exists a 1-1 function \(f: \mathbb{R} \rightarrow \mathbb{N}\).

5. Let
   \[
   A = \left\{ \frac{n - 1}{n} : n \in \mathbb{N} \right\}.
   \]
   Give a careful proof that \(\sup A = 1\). Use only the definition of supremum and the elementary properties of inequalities; avoid using any results about sequences.