1. Complete the following definitions:
   (a) Let \( f : A \to \mathbb{R} \). We say \( f \) is uniformly continuous if \( \ldots \)
   (b) Let \((x_n)\) be a sequence. We say \( x_n \to -\infty \), or \((x_n)\) diverges to \(-\infty\), if \( \ldots \)

2. State the following theorems:
   (a) Bolzano-Weierstrass Theorem.
   (b) Mean Value Theorem.

3. Give an example of a sequence \((f_n)\) of integrable functions defined on \([0, 1]\) such that \(f_n \to 0\) pointwise and \(\int_0^1 f_n \neq 0\). Be sure to give reasons for all your steps!

4. (a) Define \( g : \mathbb{R} \to \mathbb{R} \) by
   \[
g(x) = \begin{cases} 
   \sin \frac{1}{x} & \text{if } x \neq 0 \\
   0 & \text{if } x = 0.
   \end{cases}
   \]
   Prove that \( \lim_{x \to 0} g(x) \) does not exist.
   (b) Define \( f : \mathbb{R} \to \mathbb{R} \) by
   \[
f(x) = \begin{cases} 
   x \sin \frac{1}{x} & \text{if } x \neq 0 \\
   0 & \text{if } x = 0.
   \end{cases}
   \]
   Prove that \( f \) is not differentiable at 0.

5. (a) Let \( \sum_{n=0}^{\infty} c_n x^n \) be a power series. Suppose the limit
   \[
   L := \lim_{n \to \infty} \left| \frac{c_{n+1}}{c_n} \right|
   \]
   exists (and for simplicity suppose \( L \) is a positive real number). Prove that the radius of convergence of the power series is
   \[
   \frac{1}{L}.
   \]
   (b) Use the result of part (a) to find the radius of convergence of the power series
   \[
   \sum_{n=0}^{\infty} \frac{x^n}{n + e^n}.
   \]

6. Let \( f : (0, \infty) \to \mathbb{R} \) be continuous. Suppose that for all \( M > 0 \) there exist \( a, b > M \) such that \( f(a) > 4 \) and \( f(b) < 2 \).
   (a) Prove that for all \( M > 0 \) there exists \( c > M \) such that \( f(c) = 3 \). Be sure to give complete reasons for all your steps!
   (b) Use the result of part (a) to prove that there exists a sequence \((x_n)\) such that \( x_n \to \infty \) and for all \( n \in \mathbb{N} \) we have \( f(x_n) = 3 \).