MAT 371 TEST 2

INSTRUCTOR: JOHN QUIGG

Time limit: 1.5 hours (90 minutes).
No notes, books, or calculators.
This exam has 1 page.
See other instructions at bottom of this page.

1. Complete the following definitions:
   (a) If \( f: A \to \mathbb{R} \) and \( x \in A \), then \( f \) is continuous at \( x \) if 
   (b) If \( f: A \to \mathbb{R} \), then \( f \) is uniformly continuous if 

2. Let \( f: \mathbb{R} \to \mathbb{R} \), and assume \( f''(x) \) exists for all \( x \in \mathbb{R} \). Suppose
   \[ f(0) = f'(0) = f(1) = 0. \]
   Prove that there exists \( c \in (0, 1) \) such that \( f''(c) = 0 \). Hint: Mean Value Theorem.

3. Find constants \( a_0, a_1, a_2 \) such that for all \( x \in (0, 2) \) there exists \( c \) between 1 and \( x \) such that
   \[ \frac{1}{x} = a_0 + a_1(x - 1) + \frac{a_2}{2}(x - 1)^2 - \frac{1}{c^4}(x - 1)^3. \]
   Hint: Taylor’s Theorem.

4. Let \( f: [0, \infty) \to (0, 1) \). Prove that there is a strictly increasing sequence \( (n_1, n_2, n_3, \ldots) \) of natural numbers such that the sequence \( (f(n_1), f(n_2), f(n_3), \ldots) \) converges.

5. Let \( f: (0, 1) \to \mathbb{R} \) be continuous. Suppose
   \[ \lim_{x \to 0^+} f(x) = -3 \quad \text{and} \quad \lim_{x \uparrow 1} f(x) = 2. \]
   Prove that there exists \( c \in (0, 1) \) such that \( f(c) = 0 \).

Date: March 11, 1999.

Write your solutions on blank paper obtained from the Testing Clerk.
Write on only one side of each sheet, leaving a 1-inch margin on all sides.
Turn in your exam, pages face up, with this exam sheet on top.
Do not staple the pages or make any other attempt to attach them.
Show all your work, and be sure to explain your reasoning.
All problems have equal credit.
Be as thorough as you can in your solutions.