1. Let \( n \in \mathbb{N} \) and define \( f : (0, \infty) \) by \( f(x) = x^n \). Note that \( f \) is strictly increasing and continuous, and \( \lim_{x \to \infty} f(x) = \infty \). Hence it follows from the Intermediate Value Theorem that \( \text{ran } f = (0, \infty) \). Thus \( f \) has an inverse \( f^{-1} : (0, \infty) \to (0, \infty) \). (You do not have to prove any of the assertions so far.)

(a) Show how the Inverse Function Theorem can be applied (and do not forget to check the hypotheses!) to show \( f^{-1} \) is differentiable at every positive number \( y \), and give a formula for the derivative of \( f^{-1} \).

(b) Show how part (a) and the general properties of derivatives can be used to deduce the Power Rule
\[
\frac{d}{dx} x^n = nx^{n-1} \quad \text{for } x > 0
\]
for rational exponents \( n \). You may assume the version of the Power Rule for integers that we have already proved, but other than that be sure your reasoning applies to all \( n \in \mathbb{Q} \). Note: of course, if \( n \) is odd all of this could be done on \( \mathbb{R} \) rather than just \( (0, \infty) \), but do not do this here.

2. Define \( f : [0, 1] \to \mathbb{R} \) by
\[
f(x) = \begin{cases} 
1 & \text{if } x = 0 \\
0 & \text{if } 0 < x \leq 1.
\end{cases}
\]
Prove that \( f \) is integrable, and find \( \int_0^1 f \).

3. Let \( f : [a, b] \to \mathbb{R} \) be increasing.

(a) Prove that for every partition \( P = \{x_i\} \) of \( [a, b] \),
\[
M_i = f(x_i) \quad \text{and} \quad m_i = f(x_{i-1}).
\]

(b) Use part (a) to prove that \( f \) is integrable on \( [a, b] \). Hint: consider partitions with equally spaced points.